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Stability in a Network Economy:
The Role of Institutions

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Stability in a Network Economy: The Role of Institutions*

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Abstract

We consider an economy in which agents are embedded in a network of potential value-generating relationships. Agents are assumed to be able to participate in three types of economic interactions: Autarkic self-provision; bilateral interaction; and multilateral collaboration.

We introduce two stability concepts and provide sufficient and necessary conditions on the network structure that guarantee existence, both in the absence of externalities from cooperation as well as under crowding conditions. We show that institutions such as socioeconomic roles and hierarchical leadership structures are necessary for stability. In particular, the stability of more complex economic outcomes requires more stringent restrictions on the underlying network which imply more complex institutional rules that govern economic interactions. Thus, we provide support for the theory of co-evolution of institutions and economic outcomes.

Keywords: Network economies; Economic outcomes; Stability

JEL code: C72, D71, D85

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1 Equilibrium in a network economy

Stability is universally accepted as a desirable feature in system analysis. For economists, stability implies not only predictability, but also gains in happiness through the reduction of uncertainty and risk[1], and is generally seen as being conducive to economic growth. For example, Mobarak (2005) provides empirical evidence on the relation between (in)stability and economic growth. As such, the institutional arrangements studied here can be seen as promoters of economic development and growth. Instability, on the other hand, is manifested in a dysfunctional institutional organisation of the economy.

Therefore, stability signifies a well-functioning institutional setting in which more complex forms of interactions can emerge. Logically, this leads to the conclusion that there is a natural progression from simpler to more complex situations: We develop our analysis from the perspective that the emergence of more complex economic systems is only possible if there is sustained development that first occurs in more primitive institutional outcomes. For example, an anonymous stock market would not have been possible, had it not been for barter based on mutually respected property rights at an earlier stage of economic evolution.

For a stable outcome of economic interactions to emerge, our study shows that a specific institution which restricts interactions among agents is necessary and sometimes sufficient. Institutions identified here as necessary for the stability of our stylised economy are thus fulfilling a mechanistic role in the evolution of human organisation as discussed by Stoelhorst and Richerson (2013). Our analysis is also in accord with Kaufman's (2003) theory of economic organisation that builds on the writings of John R. Commons. These authors postulate the necessity of institutional rules which prescribe the domains of decision making. Here we provide a formal proof that such rules are both necessary and sufficient to ensure stability.

We furthermore see our work as complementary to studies of the co-evolution and endogeneity of culture (social rules) and economic activities (Frederking, 2002; Kuran, 2009). In our formal theory, we de-couple the stabilising function of institutional rules from the content of the economic activities. Thus, we not only gain more universal applicability, but we also identify institutional rules which function robustly in a changing environment of economic activities, albeit within the domain of the type of economic outcomes on which we focus here.

Turning to our formal model, we consider an economy consisting of economic agents who are embedded in a network of potential value-generating relationships. The generated gains from interaction are modelled as (hedonic) utility values over the possible economic activities in which these agents can engage as prescribed by the network. The restrictions implied by the network are interpreted as institutional rules that govern the underlying

[1]For a discussion on the link between happiness and risk see Dehejia, DeLeire, and Luttment (2007).

engagement of agents through the prescribed economic activities.

We use straightforward extensions of standard stability notions from matching theory (Roth and Sotomayor, 1990) and network formation theory (Jackson and Wolinsky, 1996) to define stability in two stylised economic systems. First, we consider a network *exchange economy* that is founded on bilateral interactions only. Subsequently, we extend our setting to include *multilateral* interactions, where individuals can engage with multiple partners. Such multilateral interactions are akin to multi-sided platforms as considered by Hagiu and Wright (2011) and Evans and Schmalensee (2013) in the context of market theory, extending the seminal work by Rochet and Tirole (2003) and Evans (2003).[2]

We subsequently identify conditions on the network structure underlying the economy that guarantee the existence of stable bilateral and multilateral outcomes, respectively. These conditions clearly point to institutional features of the underlying network of potential relationships as representing the social capital instilled in these networks (Portes, 1998; Dasgupta, 2005). In particular, in the case of bilateral economic outcomes, we identify the necessity of the presence of two socioeconomic roles such that all economic activities are restricted to occur between agents of distinct roles. From this viewpoint, institutional functionality is more closely related to a development process based on the deepening of the social division of labour, in the sense of Smith (1776) and his predecessors (Sun, 2012).

Regarding the stability of multilateral economic outcomes, we impose the absence of certain cycles in the underlying network which correspond to the implementation of certain social hierarchies in the represented society. Therefore, macroeconomic properties—described by these rules of social authority and hierarchy—are not simply aggregates of microeconomic features, but are "emergent" at the level of the social, institutional governance system in the economy. This interpretation is similar to the notion of *emergence* in a macro economy as put forward by Wagner (2012). Indeed, Wagner's contention is that such institutional rules of economic conduct and interaction are *explanatorily irreducible* in the sense that these rules cannot be devolved to the level of the individual decision makers in the economy.

It is worth pointing out that here we depart from other game-theoretic approaches to the study of stability of institutions that take a dynamic (Goyal and Janssen, 1995) or evolutionary (Sugden, 1995) approach. Our work, instead, treats institutional arrangements as facilitators of economic activity: They provide a well-founded environment in which such activities emerge. Here stability is treated as an intrinsic property of the topology of economic opportunity.

Finally, we distinguish our work from the transaction costs literature, e.g., Coase (1937); North and Thomas (1973); Williamson (1975); North (1990); and Greif (2006), where institutions are usually understood as devices that lower market transaction costs. Lower trans-

[2] For a review of this theory we also refer to Rochet and Tirole (2006).

action costs in turn result into increased market efficiency and consequently economic growth and development. Our approach takes these institutions as fitting specifications of underlying network properties and brings out their functional role as stabilisers of economic activity.

To show the fundamental principles of our approach, we present some simple examples and debate the concepts that are required to describe the endogenous emergence of stable economic outcomes.

1.1 An example: Bilateral barter in a network economy

We first consider a simple example of the most primitive economic outcome—that of bilateral exchange or barter. Principally, economic agents can decide to enter in an economic exchange with a potential partner. Thus, the society is endowed with a set of potential barter relationships that can be activated under mutual consent.

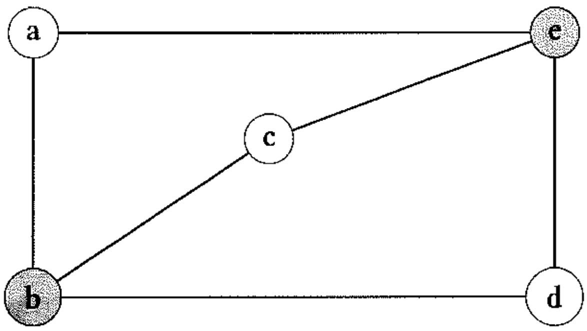

Figure 1: Network structure $\mathcal{A}$

Figure 1 depicts a network of *potential* barter relationships between five economic agents $N = \{a, b, c, d, e\}$. The network structure $\mathcal{A}$ depicts—disregarding the colouring of the nodes—the pairs of agents that can potentially engage in a value-generating relationship. Here, agents a and c cannot engage, whereas agents a and b can.

The benefits that agents receive from bartering are summarised in their hedonic utility functions defined over all potential engagements. For agent a, for example, $u_a(ab)$ represents the utility that agent a gets from bartering with b. This methodology allows us to reduce the analysis of the formation of economic outcomes to a single dimension, expressed through the hedonic utility functions of the various economic agents.[3] One can

[3]To preserve our focus on the issue of stability, we thus abstract away from the actual content of economic interactions and use, instead, hedonic utilities. The notion of hedonic games in the context of coalition formation was seminally introduced by Drèze and Greenberg (1980) and further studied by Bogomolnaia and Jackson (2004), Banerjee, Konishi, and Sonmez (2001), and Pápai (2004), among others. We point out that what distinguishes our work from those studying coalition formation games is that we employ a network approach. This application of hedonic utilities is a standard technique from the theory of clubs as well (Scotchmer, 2002).

also treat these hedonic utilities as being generated by an exchange of goods (Howitt and Clower, 2000), gifts (Akerlof, 1982), or favours (Neilson, 1999).

Returning to the example, we assume the generated hedonic utilities as follows:

	a	**b**	**c**	**d**	**e**
a	0	2	–	–	1
b	1	0	2	2	–
c	–	1	0	–	2
d	–	1	–	0	2
e	3	–	1	1	0

This matrix represents all potential barter values generated in the given network.[4]

We investigate a standard notion of stability: Every agent can participate in *at most one* exchange relationship, forming a so-called *bilateral barter outcome*. A bilateral outcome is *stable* if (i) there is no agent who prefers to remain in autarky rather than barter through the proposed pattern ("individual rationality"); and (ii) there is no pair of agents who prefer to engage in barter rather than bartering with their assigned partners ("pairwise stability").

For the values given in this example there emerge two stable bilateral barter outcomes: $\pi_1 = \{\mathbf{ae}, \mathbf{bc}, \mathbf{dd}\}$ and $\pi_2 = \{\mathbf{ae}, \mathbf{bd}, \mathbf{cc}\}$.[5]

Identifying an institutional cause of instability. Next we modify the structure of potential relationships on N as depicted in Figure 2. It is clear that agent **c** now occupies a centralised position and can interact with any of the four other agents.

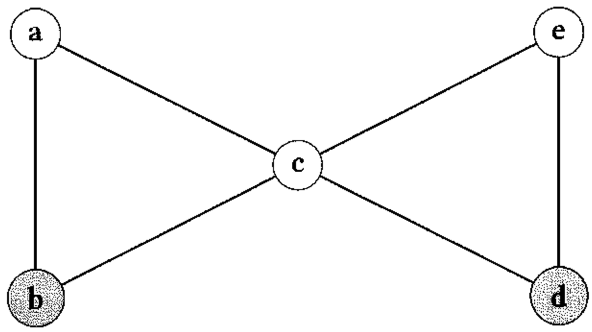

Figure 2: Network structure $\mathcal{B}$

As before, we report the hedonic values of all potential feasible barters in network $\mathcal{B}$ as an incidence matrix:

[4]The matrix is actually the *incidence matrix* of network structure $\mathcal{A}$ in which potential payoffs are reported instead of an indicator of connectedness. The number reported in field (i,j) is $u_i(ij)$. Similarly, the field (j,i) reports $u_j(ij)$. If no relationship can be formed, no payoff is reported, indicated by "–". Note that we have normalised the hedonic value of autarky to be 0, i.e. for all $i \in N$, $u_i(ii) = 0$.

[5]In fact, our main existence result stated as Theorem 3.5 implies that for any distribution of hedonic values in network structure $\mathcal{A}$ there exists a stable bilateral barter outcome that satisfies (i) and (ii) formulated here.

	a	b	c	d	e
a	0	2	1	–	–
b	1	0	2	–	–
c	2	1	0	2	1
d	–	–	1	0	2
e	–	–	2	1	0

We now claim that for these given values there does not exist a stable bilateral barter outcome in $\mathcal{B}$. Indeed, consider the outcome $\pi' = \{\mathbf{ab}, \mathbf{cd}, \mathbf{ee}\}$, then both agents **d** and **e** would prefer to barter rather than being engaged with **c** and being autarkic, respectively. Other barter patterns can be shown to be unstable as well.

What makes network structure $\mathcal{B}$ more prone to instability than network structure $\mathcal{A}$? We identify (Theorem 3.5) that the unique feature of a network economy assigning agents to two distinct economic roles—dark grey versus white nodes in Figure 1—such that interaction potentially can only occur between any two agents of distinct colours or "roles".

Clearly, structure $\mathcal{B}$ requires three distinct colours or roles, indicated as dark grey, light grey and white. The institutional binary division ensures the stability in a structure like $\mathcal{A}$, and conversely, the impossibility of guaranteed stability in a structure like $\mathcal{B}$. Indeed, with more than two roles, Condorcet cycles can be constructed to prevent the stabilising of barter patterns. Thus, stability in these simple network barter economies is founded on the property that the underlying network structure has an institutional foundation founded on exactly two socio-economic roles.

1.2 Introducing multilateral interaction

Subsequently, we look at economic agents engaging in multilateral interaction. These multilateral activities can be understood as multi-sided relational exchange platforms (Hagiu and Wright, 2011) or, simply, as "local" markets.

In our network setting, a multilateral interaction requires the active involvement of a *middleman*, who brings together the group of economic agents that forms a local market. The middleman may be seen as intermediating, coordinating, or managing the economic interaction between at least two other agents. It is important to note that which agent assumes the role of middleman is endogenous in our framework. Similarly to the bilateral case, discussed above, in this multilateral network economy, a middleman can only engage with other agents if there exist potential relationships between them. Furthermore, we assume that the economic values generated in these multilateral exchanges are again expressed as hedonic utilities.

We thus arrive at a network economy in which economic agents can engage into three types of economic activities: Autarkic self-provision; bilateral exchange; and multilateral barter through the intermediation of a middleman. Each of these three types of activities generates different hedonic utility levels for its participants. We explicitly assume that there are no widespread externalities among the various distinct bartering activities; the generated values are solely the outcome of pairwise interaction.[6]

Returning to the example of network patterns $\mathcal{A}$ and $\mathcal{B}$, we introduce a multilateral barter as any star-structured subnetwork of the imposed network. Thus, in structure $\mathcal{A}$ agent **a** can interact with **b** and **e**, while agent **b** could principally interact with **a, c** and **d.** The middleman is now the agent in the centre of the star-structured subnetwork.

Under the strict hypothesis of *no externalities* a middleman's hedonic utility in a multilateral barter is the sum of all bilateral barters in which she engages. Thus, if a is the middleman of $\mathbf{abe} = \{\mathrm{a,b,e}\}$ in structure $\mathcal{A}$, then she receives $u_a(\mathbf{abe}) = u_a(\mathbf{ab}) + u_a(\mathbf{ae}) = 3$. Any other agent participating in the multilateral exchange obtains hedonic utility equal to the utility generated in the bilateral exchange with the middleman. Thus, $u_b(\mathbf{abe}) = u_b(\mathbf{ab}) = 1$.

We again devise a standard equilibrium concept in which each agent participates in exactly one barter activity, being autarky; bilateral bartering; or multilateral bartering. In equilibrium, no agent has an incentive to join another potentially accessible activity. Such an equilibrium is called a *stable multilateral outcome.*

In structure $\mathcal{A}$ as depicted in Figure 1 there is no such stable multilateral outcomes. Indeed, take $\{\mathbf{ab},\mathbf{ecd}\}$, then agents **a** and **b** engage in pairwise barter and obtain $u_a(\mathbf{ab}) = 2$ and $u_b(\mathbf{ab}) = 1$, respectively. On the other hand, agent **e** as a middleman receives $u_e(\mathbf{ecd}) = 2$, while (regular) members **c** and **d** receive $u_c(\mathbf{ecd}) = u_d(\mathbf{ecd}) = 2$. Now, agents **a** and **e** can mutually improve their positions and agent **b** will not suffer by engaging in multilateral barter **abe**, where **a** acts as its middleman. Indeed, $u_a(\mathbf{abe}) = 3 > 2 = u_a(\mathbf{ab})$, $u_e(\mathbf{abe}) = 3 > 2 = u_e(\mathbf{ecd})$ and $u_b(\mathbf{abe}) = u_b(\mathbf{ab}) = 1$. Similarly, one can show that in all other bartering patterns there will be a profitable deviation, showing intrinsic instability.

On the other hand, contrary to the bilateral case, in structure $\mathcal{B}$ depicted in Figure 2 there now exists a stable multilateral outcome, namely, the all-inclusive collaboration $\{\mathbf{cabde}\}$ centred on middleman **c**. Here, $u_c(\mathbf{cabde}) = 6$, $u_a(\mathbf{cabde}) = u_d(\mathbf{cabde}) = 1$ and $u_b(\mathbf{cabde}) = u_e(\mathbf{cabde}) = 2$. Now, agent **a** would rather engage in a bilateral barter with **b**, but agent **b** would not agree due to the lowering of her payoff. We show in Theorem 4.9 that in fact for any payoff structure without externalities there exists a stable multilateral outcome in structures of the $\mathcal{B}$-type.

It is clear from the discussion above that, as in the bilateral case, multilateral stability is related to the institutional rules on the activation of value-generating relationships embedded in the given network of permissible relations. Our main existence theorems (Theorems 4.7 and 4.9) exactly determine these institutional conditions. In its full development, we

[6]This does not, however, in general exclude various forms of externalities among the members of a multilateral interaction.

consider different forms of stability that implement certain features of multi-agent collaboration. We distinguish "open" from "closed" multilateral platforms: In the latter a middleman fully controls the admittance of agents, while in the former this control is limited. Openness allows for the implementation of certain external effects. Closedness can only be considered in the complete absence of such externalities.

The remainder of this paper is organised as follows. Section 2 introduces our relational approach to economic interaction. Section 3 discusses stability in bilateral economies, while Section 4 extends our analysis to include multilateral platforms. In this setting we analyse the emergence of stable multilateral outcomes if there are no externalities and discuss the implications of the introduction of certain size-based externalities. We also discuss how hierarchical structures might function as institutional guides to avoid instability. Proofs of the main theorems are collected in three appendices.

2 A relational approach to economic interaction

In this section we introduce some fundamental concepts from social network analysis[7] allowing us to develop key concepts in our relational approach to describing networked economic activities.

The main postulate of our theoretical construct is that all economic interaction is principally relational. Therefore, an *economic activity* is abstractly defined as any interaction between a group of linked agents that generates a hedonic utility value for each of its participants (Granovetter, 2005). We emphasise that from this perspective the economy solely consists of relational activities.[8]

Autarkic activities. Throughout we work with a finite set of *economic agents* denoted by $N = \{1, \dots, n\}$. These economic agents can engage in three different relational economic activities that generate individual economic values to the participants. The first and most primitive form of economic activity is that of *economic autarky*, in which an agent $i \in N$ engages in home production only. For an individual economic agent $i \in N$ we denote by ii the agent's possibility to engage self-sufficiently. Thus, we arrive at the *class of all autarkic activities* that we denote by $\Omega = \{ii \mid i \in N\}$. The assigned hedonic utility level $u_i(ii)$ to an agent $i \in N$ in autarky is interpreted as the generated subsistence level for that agent.

[7]For a comprehensive overview of concepts from network analysis and network formation theories, we refer to Jackson and Wolinsky (1996), Jackson (2003), Jackson (2008) and Newman (2010).

[8]Therefore, within this framework a market is viewed as a value-generating cooperative activity. But a market is local rather than global and anonymous; it is only open to its members, where potential membership is determined by the underlying structure of potential trade relationships.

Bilateral interactions. A second type of economic activity is that of a *bilateral interaction* in the sense that two agents i and j engage into some bilateral activity such as commodity exchange or service provision for monetary compensation that generates hedonic utility values for both of these agents.

Formally, consider any pair of agents $i, j \in N$ with $i \neq j$. The mathematical expression $ij = \{i, j\}$ now represents the bilateral economic activity involving agents i and j.[9] We assume that there are restrictions on the possible bilateral interactions conforming to a set of institutional rules. A network structure Γ now exactly represents the institutionally allowable, that is, feasible, bilateral interactions between agents in the population N:

$$\Gamma \subseteq \Gamma_N = \{ij \mid i, j \in N \text{ and } i \neq j\}. \tag{1}$$

Throughout we assume that for every agent $i \in N$ there is some $j \in N$ with $ij \in \Gamma$.

In terms of our framework one can think of the pair (Ω, Γ) as the institutional matrix in which all economic activities emerge. These autarkic and bilateral activities are called *simple* and $\Delta^m = \Omega \cup \Gamma$ is now referred to as a *simple interaction structure.*

For any sub-structure $H \subseteq \Delta^m = \Omega \cup \Gamma$ we denote

$$N(H) = \{i \in N \mid \text{There is some } j \neq i \text{ such that } ij \in H\} \tag{2}$$

as the set of economic agents that are bilaterally engaged within the sub-structure H. It is easy to see that $N(H) = N(H \setminus \Omega)$. Also, for every $H \subseteq \Gamma$, if $H \neq \varnothing$, then $N(H) \neq \varnothing$. Finally, due to the assumptions made, it holds that $N(\Delta^m) = N(\Gamma) = N$.

We define a *path* between any two distinct agents $i \in N$ and $j \in N$ in $H \subseteq \Gamma$ as a sequence of distinct agents $P_{ij}(H) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = j$, $i_k \in N$ and $i_k i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. We define a *cycle* in H to be a path of an agent from herself to herself which contains at least two other distinct agents, *i.e.*, a cycle in H from i to herself is a path $C = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = i$, $m \geqslant 4$, $i_k \in N$, and $i_k i_{k+1} \in H$ for all $k \in \{1, \ldots, m-1\}$. The length of the cycle C is denoted by $\ell(C) = m - 1 \geqslant 3$. A sub-structure $H \subseteq \Gamma$ is called *acyclic* if H does not contain any cycles.

Agent i's *neighbourhood* in sub-structure H is defined as $N_i(H) = \{j \in N \mid ij \in H\}$. Note here that if $i \in N_i(H)$, then $ii \in H$. Also, by the definition of the bilateral interaction structure Γ, it holds that $N_i(\Gamma) \neq \varnothing$ for any $i \in N$. We can also express the neighbourhood of an agent within an arbitrary structure $H \subseteq \Delta^m$ in terms of its link based analogue, *i.e.*, $L_i(H) = \{ij \in H \mid j \in N_i(H)\} \subseteq H$. Therefore, $L_i(\Delta^m) = \{ii\} \cup L_i(\Gamma)$ is the set of feasible simple activities that i can potentially participate in.

[9] We remark here that $ij = ji$. Note that if $i = j$, the relational activity ii represents again the economic autarky of agent i.

Multilateral interactions. Extending the setting of a simple interaction structure (Ω, Γ) we introduce a third type of relational economic activity, that of a multilateral interaction. Such complex activities are assumed to be centred around a "middleman", representing an agent who acts as a hub in the network structure of this activity. In particular, a middleman brings together a number of economic agents with whom she already has an established economic bilateral relationship. This is formalised as follows:

Definition 2.1 *Let* $\Gamma \subseteq \Gamma_N$ *be a network structure on* N.
A ***multilateral interaction*** *in* Γ *is a sub-structure* $G \subseteq \Gamma$ *such that* $|N(G)| \geqslant 3$ *and there is a unique agent* $i \in N(G)$ *such that* $N_i(G) = N(G) \setminus \{i\}$ *and that for all other agents* $j \in N(G) \setminus \{i\}$ *it holds that* $N_j(G) = \{i\}$. *The agent* i *is called the* ***middleman*** *of the multilateral interaction* G, *denoted by* $\mathcal{K}(G) \in N(G)$.

Thus, a multilateral interaction has at least three members. Furthermore, a multilateral interaction has an explicit star structure in Γ. This implies that a multilateral interaction has a relational centre, representing the middleman, binding and coordinating all constituting bilateral relations of this complex activity.

A multilateral interaction $G \subseteq \Gamma$ might also be interpreted as a mathematical expression of a multi-sided interaction *platform* provided by its middleman $\mathcal{K}(G)$ in the sense of Hagiu and Wright (2011).[10] However, it is clear that our abstract concepts allow more broad interpretations such as clubs (Buchanan, 1965) or local authorities (Tiebout, 1956).

Using this definition of multilateral interactions, we can introduce some auxiliary concepts and notation.

Definition 2.2 *Let* $\Gamma \subseteq \Gamma_N$ *be some network structure. The collection of all feasible multilateral interactions is now given by*

$$\Sigma(\Gamma) = \{ G \mid G \subseteq \Gamma \text{ is a multilateral interaction in } \Gamma \} \tag{3}$$

$\Sigma(\Gamma)$ *is called the* ***multilateral structure*** *on* Γ.
The triple $(\Omega, \Gamma, \Sigma(\Gamma))$ *is referred to as a* ***feasible activity structure*** *on* N *consisting of all autarkies* $G_1 \in \Omega$, *all feasible bilateral interactions* $G_2 \in \Gamma$, *and all feasible multilateral interactions* $G_3 \in \Sigma(\Gamma)$.[11]

We interpret the feasible activity structure $(\Omega, \Gamma, \Sigma(\Gamma))$ as a mathematical representation of all institutionally feasible interactions in the economy. This structure acts as a representation of the institutional rules that govern the interaction in the economy as a whole.

[10] Hagiu and Wright (2011, page 7) define a multi-sided platform as "an organization that creates value primarily by enabling direct interactions between two (or more) distinct types of affiliated customers".

[11] The union of a feasible activity structure, $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$, serves as an alternative description for this feasibility structure.

Finally, we remark that we can now introduce the *set of middlemen* in Γ as the collective of middlemen of all multilateral activities in $\Sigma(\Gamma)$:

$$\mathcal{K}(\Gamma) = \{i \in N \mid i = \mathcal{K}(G) \text{ for some } G \in \Sigma(\Gamma)\}. \tag{4}$$

It is clear that agents in $\mathcal{K}(\Gamma) \subset N$ play a crucial role in the formation of interaction structures in the economy. These agents represent therefore a class of potential *entrepreneurs* in the economy.

3 Stability in bilateral network economies

Here we discuss stability in an economy with autarkic and bilateral interactions only, extending the model of a matching economy introduced in Gilles, Lazarova, and Ruys (2007).

Throughout we assume that every individual $i \in N$ has complete and transitive preferences over her set of feasible simple activities $L_i(\Delta^m) = \{ii\} \cup L_i(\Gamma) \subseteq \Delta^m = \Omega \cup \Gamma$ in which she can engage. We represent these preferences by a *hedonic utility function* $u_i^m : L_i(\Delta^m) \to \mathbb{R}$. Let $u^m = (u_1^m, \dots, u_n^m)$ denote the resulting *hedonic utility profile.*

Definition 3.1 *A* ***bilateral economy*** *is defined as a triple* $\mathbb{E}^m = (N, \Delta^m, u^m)$ *in which* N *is a finite set of individuals,* $\Delta^m = \Omega \cup \Gamma$ *is a simple activity structure on* N*, and* $u_i^m : L_i(\Delta^m) \to \mathbb{R}$, $i \in N$*, is a hedonic utility profile on* Δ^m.

The main hypothesis in our definition of stability in a bilateral economy is that each individual $i \in N$ activates *exactly one* of her activities in $L_i(\Delta^m)$.

Definition 3.2 *An* ***outcome*** *in the bilateral economy* $\mathbb{E}^m = (N, \Delta^m, u^m)$ *is a mapping* $\pi : N \to \Delta^m$ *such that*

(i) $\pi(i) \in L_i(\Delta^m)$ *for all* $i \in N$ *and*

(ii) $\pi(i) = ij$ *implies that* $\pi(j) = ij$ *for all* $i, j \in N$.

We refer to outcomes in a bilateral economy as *bilateral outcomes.* A bilateral outcome π can equivalently be represented by the induced sub-structure in Δ^m

$$\pi(N) = \{\pi(i) \mid i \in N\}. \tag{5}$$

The set of all bilateral outcomes π in $\mathbb{E}^m$ is denoted by Π^m. We remark that by the imposed hypotheses and definitions, $\Pi^m \neq \emptyset$. In particular, $\Omega \in \Pi^m$ and, according to the assumptions made on Γ, for any agent $i \in N$, there exist some $\pi \in \Pi^m$ with $\pi(i) = ij$ for every $ij \in \Gamma$.

With slight abuse of notation, we use $u_i^m(\pi)$ to denote the hedonic utility that agent $i \in N$ receives under outcome $\pi \in \Pi^m$, *i.e.*, $u_i^m(\pi) = u_i^m(\pi(i))$.

We apply the standard assumptions of individual rationality and a no-blocking condition from matching theory—denoted here as "pairwise stability" (Roth and Sotomayor, 1990; Jackson and Wolinsky, 1996)—to define a notion of stability in bilateral economies.

Definition 3.3 *An outcome $\pi \in \Pi^m$ is **stable** in the bilateral exchange economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ if all bilateral interactions generated by π satisfy the following properties:*

Individual Rationality (IR): $u_i^m(\pi) \geqslant u_i^m(ii)$ *for all $i \in N$, and;*

Pairwise stability (PS): *There is no blocking bilateral interaction with regard to π, in the sense that for all $i, j \in N$ with $i \neq j$, $ij \in \Gamma$ and $\pi(i) \neq ij$ it holds that*

$$u_i^m(ij) > u_i^m(\pi) \text{ implies that } u_j^m(ij) \leqslant u_j^m(\pi). \tag{6}$$

For an economy to have persistent access to gains from organisation, its social structure has to *universally* admit stable outcomes. Hence, regardless what productive capabilities and consumption preferences the individual economic agents hold—both represented here by their (hedonic) utility functions—a stable outcome has to exist in the corresponding bilateral economy.

Definition 3.4 *A network structure Γ on N **supports universal bilateral stability** if for every hedonic utility profile u^m on $\Delta^m = \Omega \cup \Gamma$ there exists at least one stable bilateral outcome in the bilateral exchange economy $\mathbb{E}^m = (N, \Delta^m, u^m)$.*

Clearly, a network structure that supports universal bilateral stability implies that the institutional organisation of the economy supports stability regardless of the exact individual preferences. In this regard, such a structure reflects institutional features which promote and enhance the emergence of stable patterns of economic activities in a decentralised manner.

The next result identifies the necessary and sufficient conditions for universal bilateral stability. Similar conditions have already been established in the literature on matching markets.

Theorem 3.5 *A network structure Γ on N supports universal bilateral stability if and only if Γ is bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that*

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{7}$$

For a proof of this result we refer to Appendix A of the paper.

Theorem 3.5 has a clear interpretation. Any feasible activity structure that supports universal bilateral stability has to be based on two clearly defined and separable socioeconomic roles such that interactions can only occur between pairs of agents of distinct roles. The example of the bilateral economy in Section 1.1 illustrates this.

4 Stability in multilateral network economies

Next we extend the scope of our analysis to include multilateral interactions in $\Sigma(\Gamma)$. Let $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ be a feasible activity structure on the population N. For $i \in N$ we introduce the set of all feasible activities in which agent i can participate as

$$\mathcal{A}_i(\Delta) = \{ii\} \cup \{ij \mid ij \in \Gamma\} \cup \{G \mid G \in \Sigma(\Gamma) \text{ and } i \in N(G)\}. \tag{8}$$

We denote by $\mathcal{A}(\Delta) = \cup_{i \in N} \mathcal{A}_i(\Delta)$ the collection of all feasible activities available to all agents in the economy.

For every economic agent $i \in N$, her preferences are now represented as a hedonic utility function $u_i : \mathcal{A}_i(\Delta) \to \mathbb{R}$. Let $u = (u_1, \dots, u_n)$ be a profile of hedonic utility functions for all agents in N. Let $\mathcal{U}$ be the class of all hedonic utility profiles on Δ.

Now a *network economy* is defined to be a structure Δ of feasible activities—autarky, bilateral, as well as multilateral—and a hedonic utility function profile:

Definition 4.1 *A* ***network economy*** *is a triple* $\mathbb{E} = (N, \Delta, u)$ *in which* N *is a finite set of economic agents,* $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ *is a feasible activity structure, and* $u \in \mathcal{U}$ *is a profile of hedonic utility functions* $u_i : \mathcal{A}_i(\Delta) \to \mathbb{R}$ *for every* $i \in N$*.*

As in the case with bilateral economies, here it is assumed that agents participate in exactly one activity.

Definition 4.2 *Let* $\mathbb{E} = (N, \Delta, u)$ *be a network economy. A* ***multilateral outcome*** *in* $\mathbb{E}$ *is a mapping* $\lambda : N \to \mathcal{A}(\Delta)$ *such that* $\lambda(i) \in \mathcal{A}_i(\Delta)$ *and* $\lambda(i) = G \in \Delta$ *implies that* $\lambda(j) = G$ *for all* $j \in N(G)$*.*
A multilateral outcome λ *can also be represented by its corresponding partitioning given by* $\Lambda = (G_1, \dots, G_m) \equiv \lambda(N) \subseteq \Delta$*.*

A multilateral outcome is defined to be stable if it satisfies certain standard stability conditions from matching theory (Roth and Sotomayor, 1990), network formation theory (Jackson and Wolinsky, 1996), and Tiebout equilibrium theory for club economies (Gilles and Scotchmer, 1997).

For network economies we introduce two notions of stability reflecting two types of assumed control of middlemen over the membership of provided platforms.

Definition 4.3 *Let* $\mathbb{E} = (N, \Delta, u)$ *be a network economy.*

(a) *A multilateral outcome* $\lambda^\star : N \to \mathcal{A}(\Delta)$ *generating* $\Lambda^\star = (G_1^\star, \dots, G_m^\star)$ *is* ***stable*** *in the network economy* $\mathbb{E}$ *if for every* $p \in \{1, \dots, m\}$ *the activity* $G_p^\star \in \Lambda^\star$ *satisfies the* individual rationality *IR and two* pairwise stability *conditions PS and PS* as specified below:*

IR *for all* $i \in N(G_p^\star)$ *it holds that* $u_i(G_p^\star) \geqslant u_i(ii)$;

PS *for all distinct agents* $i \in N(G_p^\star)$ *and* $j \in N(G_q^\star)$ *with* $q \in \{1, \dots, m\}$ *and* $ij \in \Gamma$, $ij \notin G_p^\star \cap G_q^\star$:

$$u_i(ij) > u_i(G_p^\star) \quad \textit{implies} \quad u_j(ij) \leqslant u_j(G_q^\star); \tag{9}$$

PS* *for all distinct agents* $i \in N(G_p^\star)$ *and* $j \in N(G_q^\star)$ *with* $q \in \{1, \dots, m\}$ *with* $ij \in \Gamma$, $ij \notin G_p^\star \cap G_q^\star$ *and either* $j = \mathcal{K}(G_q^\star)$ *or* $G_q^\star \in \Gamma$:

$$u_i(G_q^\star \cup \{ij\}) > u_i(G_p^\star) \quad \textit{implies} \quad u_j(G_q^\star \cup \{ij\}) \leqslant u_j(G_q^\star). \tag{10}$$

(b) *A multilateral outcome* $\lambda^\star : N \to \mathcal{A}(\Delta)$ *generating* $\Lambda^\star = (G_1^\star, \dots, G_m^\star)$ *is* ***strongly stable*** *in the network economy* $\mathbb{E}$ *if* $\lambda^\star$ *is stable—satisfying IR, PS and PS*—in* $\mathbb{E}$ *and, additionally, for every* $p \in \{1, \dots, m\}$ *the activity* $G_p^\star \in \Lambda^\star$ *satisfies* Reduction Proofness *[RP]:*

RP *If* $G_p^\star$ *is a multilateral interaction, i.e.,* $G_p^\star \in \Sigma(\Gamma) \cap \Lambda^\star$, *it holds that for every sub-structure* $G \subseteq G_p^\star$

$$u_i(G) \leqslant u_i(G_p^\star) \tag{11}$$

where $i = \mathcal{K}(G_p^\star) = \mathcal{K}(G)$ *is the middleman of both* $G_p^\star$ *and* G.

Condition IR is a standard individual rationality condition that allows an individual to opt out of an economic activity if she is better in autarky. The first pairwise stability condition is adapted in similar fashion as formalised for bilateral economies in Definition 3.3. It rules out blocking opportunities for pairs of agents who are not connected to each other in the present equilibrium.

The second pairwise stability condition PS* rules out blocking opportunities for pairs of agents at least one of whom can add a link without severing his existing links in the present equilibrium. Hence, such an agent is either a middleman or involved in a bilateral interaction. This condition requires that there are no two distinct agents who want to be linked to each other in a multilateral interaction in which one of them is a middleman.[12]

[12]Note that both middlemen and agents linked in a bilateral interaction, have multiple types of blocking opportunities: such agents can add a link with or without severing their current links. Such agents are subject to both (no blocking) conditions PS and PS*.

Both PS and PS* are concerned with the formation of new links by deviating agents. These conditions do *not* allow a middleman to block access to the multilateral interaction: Stability is founded on openness of multilateral interactions in the sense that new members can join the interaction if that is to their benefit. There are numerous economic activities and platforms that satisfy the principle of openness such as trading posts (stores) and markets, open source communities, and many economic service provision cooperatives (clubs). In most of these cases, if entrants follow the house rules of the platform in question, they will not be excluded from participation.

The notion of strong stability is more demanding in the sense that Condition RP explicitly "closes" a multilateral interaction: The middleman can exclude any participant from the platform. In particular, RP states that agents are refused membership if that benefits the middleman. In economic practice we encounter many such closed complex activities and platforms. We mention as examples team production situations, exclusive clubs (guilds and unions), and particular supply chains in which an intermediary may discontinue a procurement relation with a primary input supplier in case demands of a new client render the interaction with that supplier no longer beneficial.

Under (regular) stability, a middleman is merely a *coordinator* of the multilateral interaction that is fully open to participation. On the other hand, under strong stability a middleman is rather considered to be a *manager* of the activity under consideration, since she controls agents' access. We emphasise that strong stability implies stability, *i.e.*, management implies coordination, but that the reverse is not true.

4.1 Separability: The absence of externalities

After having established a model of relational economic activities, we investigate the existence of stable multilateral outcomes. We distinguish two types of network economies: Economies with network externalities and spillovers affecting the performance of multilateral interactions and economies without such network externalities. First, we investigate economies without network externalities.

Definition 4.4 *Let* $\mathbb{E} = (N, \Delta, u)$ *be a network economy.*

(i) *The hedonic utility function* $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$ ***exhibits no externalities*** *if for all* $G_i \in \mathcal{A}_i(\Delta)$ *and* $H_i \in \mathcal{A}_i(\Delta)$ *with* $N_i(G_i) = N_i(H_i)$, *it holds that* $u_i(G_i) = u_i(H_i)$. *The class of utility profiles consisting of utility functions exhibiting no externalities is now denoted by* $\mathcal{U}_n \subset \mathcal{U}$.

(ii) *The network economy* $\mathbb{E} = (N, \Delta, u)$ *is* ***separable*** *if* $u \in \mathcal{U}_n$.

In the absence of externalities an agent derives value only from interactions with agents with whom she is linked *directly*. Thus, changes in multilateral interactions regarding third parties do not affect the hedonic utility value of a member of that interaction. Although this seems to be a very stringent condition, it is a common assumption in traditional club and Tiebout economies, where the provider of a local public good acts as a middleman in our terms.[13]

In addition to separability, we introduce the superadditivity of the hedonic utility functions. This property reflects synergies which are assumed to be attributed to the middleman who coordinates the multilateral value generation process.

Definition 4.5 *Let Γ be a network structure on N. For agent $i \in N$, the hedonic utility function $u_i \colon \mathcal{A}_i(\Delta) \to \mathbb{R}$ is* ***superadditive*** *if for any $G_i \in \mathcal{A}_i(\Delta)$ and $H_i \in \mathcal{A}_i(\Delta)$ with $G_i \cup H_i \in \mathcal{A}_i(\Delta)$ and $G_i \cap H_i = \varnothing$ it holds that $u_i(G_i \cup H_i) \geqslant u_i(G_i) + u_i(H_i)$.*

Furthermore, we say that a utility profile $u \in \mathcal{U}$ on Γ is *superadditive* if the hedonic utility function u_i is superadditive for every agent $i \in N$. The collection of all superadditive utility profiles is denoted by $\mathcal{U}_s \subset \mathcal{U}$.

Within the context of network economies we address the existence of stable multilateral outcomes for arbitrary separable and superadditive hedonic utility profiles. Formally, we introduce:

Definition 4.6 *Let Γ be a network structure and let $\mathcal{U}^\star \subseteq \mathcal{U}$ be some given class of utility profiles on Γ. Then Γ* ***supports universal (strong) multilateral stability*** *on the class $\mathcal{U}^\star$ if for every utility profile $u \in \mathcal{U}^\star$ there exists a (strongly) stable multilateral outcome $\lambda^\star$ in the network economy $\mathbb{E} = (N, \Delta, u)$, where Δ is generated by Γ.*

We denote by $\overline{\mathcal{U}} = \mathcal{U}_s \cap \mathcal{U}_n$ the class of all hedonic utility profiles that satisfy the superadditivity as well as the non-externality properties.

Theorem 4.7 *The network structure Γ supports universal strong multilateral stability on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities if and only if Γ satisfies the property that for every cycle $C \subseteq \Gamma$: $\ell(C) = 6k$, where $k \in \mathbb{N}$ is some integer.*

The proof of Theorem 4.7 is given in Appendix B.

Theorem 4.7 imposes a very strong institutional condition on the underlying network structure of the economy. Indeed, any cycle of length other than a 6-fold is excluded. In particular, there are no triads of length 3 present in the network, thus implying that the network only consists of weak ties in the sense of Granovetter (1973).

Due to the absence of such strong ties, strong stability is hard to guarantee. One particular class of social arrangements that implies the conditions in Theorem 4.7 is that of a strict social hierarchy, represented by a fully acyclic network.

[13]In this regard if all multilateral interactions exhibit such non-externalities towards its members, the activities represented are separable and, thus, can in principle be evaluated objectively. This underlies the principle of pricing membership of clubs in a club economy (Gilles and Scotchmer, 1997) as well as the Samuelson conditions in the efficient provision of a pure public good (Samuelson, 1954).

Corollary 4.8 *If the network structure Γ is acyclic, then Γ supports universally strongly stable multilateral outcomes on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities.*

Our investigation of the necessary and sufficient conditions for the support of (regular) universal stability results into the identification of a larger class of structures guaranteeing stability.

Theorem 4.9 *The network structure Γ supports universal multilateral stability on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities if and only if Γ satisfies the property that for every cycle $C \subseteq \Gamma$: $\ell(C) = 3k$, where $k \in \mathbb{N}$ is some integer.*

The proof of Theorem 4.9 is again relegated to Appendix B.

As with Theorem 4.7, we interpret the necessary and sufficient condition on the network structure stated in Theorem 4.9 as a mode of institutional governance. Interestingly, our result shows that triads—represented as cycles of length three—are not associated with inherent instability of the multilateral outcomes. Hence, certain network structures consisting of strong as well as weak ties guarantee the emergence of stability.

In particular, many sensible social hierarchies satisfy the conditions of Theorem 4.9. Here we discuss two classes of hierarchical network architectures that satisfy these conditions.

One-boss-one-partner networks. A possible intuition for Theorem 4.9 is possible through application of Burt's (1992) *tertius gaudens* principle: In the presence of tension between two agents, a third agent can take control over the relational benefits and realise her most preferred outcome. This is the case when a manager exploits the competition of two subordinates for a promotion or when a broker benefits from the tension between a buyer and a seller by extracting all gains from trade.

A class of networks reflecting this *tertius gaudens* principle satisfies the so-called *one-boss-one-partner* property.

Definition 4.10 *A network structure Γ satisfies the* ***one-boss-one-partner property****, if for each of its components there exists a mapping h of agents N into ordered hierarchical levels $\mathcal{H} = (H_1, \ldots, H_K)$, such that*

(i) *each agent i such that $h(i) = H_k$ with $k > 1$ is linked with exactly one agent from the preceding hierarchical level H_{k-1} and she has no links with agents from higher order levels H_{k-s} with $s > 2$;*

(ii) *an agent i such that $h(i) = H_k$ can have at most one link with an agent j with $h(j) = H_k$ from her own hierarchical level, and;*

(iii) *if there are two distinct agents* i, j *with* $ij \in \Gamma$ *and* $h(i) = h(j) = H_k$ *with* $k > 1$*, then there is some agent* b *with* $h(b) = H_{k-1}$ *and* $ib \in \Gamma$ *as well as* $jb \in \Gamma$.

An example of an one-boss-one-partner network is shown on Figure 3 where agents are mapped into three hierarchical levels—dark grey, light grey, and white. Here, if two agents of the same hierarchical level are linked together, like, e.g., e and f on the lowest level, they also share the same 'boss' located on the level above.

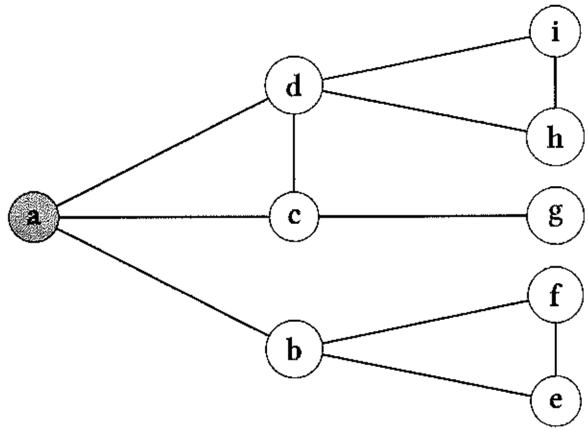

Figure 3: One-boss-one-partner network structure

Clearly, a link across hierarchical levels represents an authority relation. By taking links within the same hierarchical level to represent substitutable skills among colleagues, we identify competitive tension among co-workers. Indeed, control and tension are the key notions underlying the principle of *tertius gaudens*: The control over lower ranked individuals is reinforced by divergent preferences between them. Thus, a higher ranked individual may induce more effort and better performance from her subordinates. In Figure 3 we identify two controlled or managed branches $b - e - f$ and $c - d - g - h - i$ in the organisation described that are based on these principles.

Furthermore, one-boss-one-partner networks comply with the structural holes theory developed in Burt (1992). In particular, the institutional rules embedded in these networks comply with the principle that there should be a minimal number of non-redundant links.

On the one hand, the institutional rules introduced in one-boss-one-partner networks induce limited connectivity across branches. This, in turn, facilitates specialisation, provides opportunities for developing originality and innovation as any branch of the hierarchy can develop an independent mode of governance, and stimulates product building or information generation.

On the other hand, as stated, these institutional rules do not completely eliminate redundant links. In the presence of redundant links, the flow of information can still reach all agents in the organisation, even if some links fail. Redundant links may also provide higher speed of transmission of information along the organisation network and ensure sufficient level of compatibility across independently developing branches. In the example, one of the links among $\{ac, cd, ad\}$ is redundant, but functions as an insurance against the severance of the other links.

One-market-maker-one-partner networks. Another network architecture for which the conditions of Theorem 4.9 are satisfied, is that of a so-called *one-market-maker-one-partner* network.

Definition 4.11 *A network structure* Γ *satisfies the* ***one-market-maker-one partner property*** *if in each component there exists a mapping* $r: N \to \{A, B, C\}$*, where* A*,* B *and* C *are three distinct roles—interpreted as two partner roles (*A *and* B*) and a market-maker role (*C*)—such that*

(i) *for all* $i, j \in N$ *with* $i \neq j$ *and* $r(i) = r(j)$: $ij \notin \Gamma$;

(ii) *there is at most one market maker, i.e.,* $|C| \leq 1$*, and;*

(iii) *each A-agent has at most one link with a B-agent and vice versa.*

An example of an one-market-maker-one-partner network is the exchange network economy $\mathcal{B}$ depicted in Figure 2.

Another example of an one-market-maker-one-partner network is shown in the right panel in Figure 4. This network structure is based on a construction method using a simple binary network structure depicted in the left hand panel to which a market maker, node g, is added.[14]

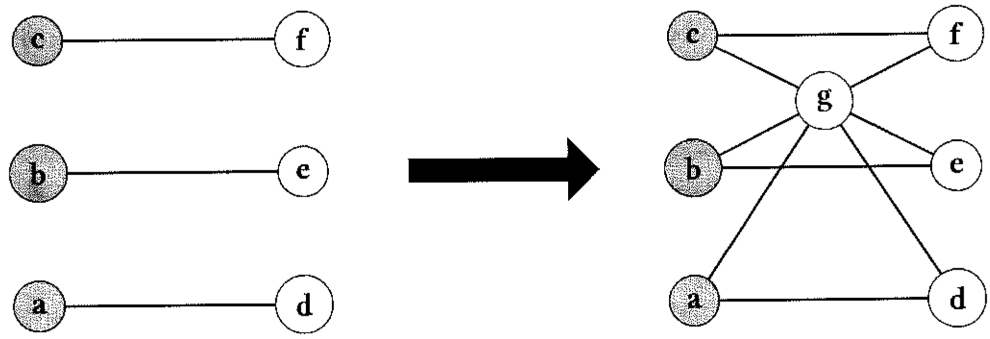

Figure 4: Construction of a one-market maker-one-partner network

Note that here the market maker g may benefit from the tension in negotiations between any given pair of gray agents to extract rent by providing outside opportunities,

[14]In the left hand panel of Figure 4 we depict a network of bilateral exchanges between two types of agents: dark grey and light grey. The implicit assumption is that the producers of each good have the possibility to invest in only one link with another producer. Clearly, in this network there are structural holes (Burt, 1992). An entrepreneurial agent can exploit the presence of structural holes and invest in the links that will connect each component, thus, creating increased opportunities for trade. This role of market maker is taken by agent g in the right-hand panel of Figure 4.

using the *tertius gaudens* principle. In the right-hand panel, each market participant has a choice of engaging directly with her potential partner or execute her trade through the market maker. It is worth pointing out that in order to ensure the existence of stability the role of the market maker cannot be contested (Gilles and Diamantaras, 2013).

Based on the above discussion we can derive the following corollary:

Corollary 4.12 *If the network structure Γ satisfies the One-boss-one-partner property, or the One-market-maker-one-partner property, then Γ supports universal multilateral stability on the class $\overline{\mathcal{U}}$ of superadditive hedonic utility profiles exhibiting no externalities.*

4.2 Introducing size-based externalities

Next we consider conditions under which stable economic equilibria in the presence of size-based externalities exist. Unfortunately, in the presence of externalities it is impossible to derive general statements similar to the ones stated in Theorems 3.5, 4.7, or 4.9. Through a series of examples we show that only certain types of externalities allow for such a general treatment.

In particular, we show that imposing strong acyclicity conditions on the network structure supports the emergence of stable multilateral outcomes under crowding. In the literature on Tiebout and club economies such crowding externalities have been investigated extensively (Conley and Wooders, 1997; Conley and Konishi, 2002).

For utility profiles with crowding, the number of agents in a multilateral interaction is determining the size of the externality. The *identity of the middleman* of the multilateral interaction determines whether crowding is positive or negative, but the identity of the membership does not affect the nature or magnitude of the crowding effect.

Definition 4.13 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy. The utility function u **exhibits a size-based externality** if for every multilateral interaction $G \in \Sigma(\Gamma)$,*

$$u_i(G) = \sum_{j \in N_i(G)} u_i(ij) + \alpha_c \cdot [\, \#N(G) - 2\,] \tag{12}$$

for all $i \in N(G)$, where $c = \mathcal{K}(G)$ is G's middleman and $\alpha_c \in \mathbb{R}$ is a middleman-specific synergy parameter.

If the middleman c of G has an externality parameter $\alpha_c > 0$, she brings about positive size-based effect. This refers to "economies to club size" based on the total size of the interaction gathered around this middleman. If, on the other hand, this middleman has an externality parameter $\alpha_c < 0$, she causes negative "crowding" (Conley and Wooders, 1997).

First we report that there exist network economies exhibiting size-based externalities in which there is no stable multilateral outcome. An example is presented below.

Example 4.14 Let $N = \{1,2,3,4\}$ and $\Gamma = \{12,23,34\}$. Let $\alpha_2 = 200$ and $\alpha_3 = -50$. Let the hedonic utility profile be such that $u_1(12) = u_2(22) = u_3(33) = -100$, $u_1(11) = u_2(12) = 0$, $u_2(23) = u_4(34) = 100$, $u_4(44) = 90$, $u_3(23) = 60$, and $u_3(34) = 300$. Using linear size-based externalities, we can compute the utility levels in the two possible multilateral interactions[15] 213 and 324 in a straightforward manner: $u_1(213) = 100$, $u_2(213) = 300$, $u_3(213) = 260$, $u_2(324) = u_4(324) = 50$, and $u_3(324) = 310$.
We now claim that in this example there is no stable multilateral outcome. First, consider the outcome $(12,34)$. It is not stable because Condition PS* is not satisfied: $50 = u_2(324) > u_2(12) = 0$ and $310 = u_3(324) > u_3(34) = 300$. Also, since $-100 = u_2(22) < u_2(324) = 50$, the Condition PS* is not satisfied for the outcome containing $(11,22,34)$. Next, consider $(11,324)$, which is not stable since IR for agent 4 is not satisfied: $50 = u_4(324) < u_4(44) = 90$. Moving on, the outcome $(11,23,44)$ is not stable due to a violation of PS*: $0 = u_1(11) < u_1(213) = 100$ and $100 = u_2(23) < u_2(213) = 300$. Finally, $(213,44)$ is not stable due to a violation of PS: $260 = u_3(213) < u_3(34) = 300$ and $90 = u_4(44) < u_4(34) = 100$. Using the same reasoning, we find that $(12,33,44)$ and $(11,22,33,44)$ are not stable either. ♦

Second, stability may not be possible even if we impose *uniform* linear size-based externalities on all middlemen. The following two examples illustrate this point. The first example imposes uniform crowding.

Example 4.15 Let $N = \{1,2,3\}$ and let $\Gamma = \{12,23\}$. Now consider $\alpha_2 = -2$. Let the utility function be such that $u_i(ii) = 0$ for all $i = 1,2,3$ and $u_1(12) = u_2(12) = 3$, $u_2(23) = 4$, and $u_3(23) = 1$. Using the linear size-based externality formulation, we compute the utility levels in the multilateral interaction 213 in a straightforward manner: $u_1(213) = 1$, $u_3(213) = -1$, and $u_2(213) = 5$. We now claim that there is no stable multilateral outcome in this network economy.
To show this, first, consider $(12,33)$. This outcome is not stable due to a violation of PS: $3 = u_2(12) < u_2(23) = 4$ and $0 = u_3(33) < u_3(23) = 1$. Similarly, $(11,22,33)$ is not stable. Next, $(11,23)$ is not stable due to a violation of PS*: $0 = u_1(11) < u_1(213) = 1$ and $4 = u_2(23) < u_2(213) = 5$. Finally, (213) is not stable due to a violation of IR for agent 3: $-1 = u_3(213) < u_3(33) = 0$. ♦

Finally, we consider a 5-agent circular network structure. Here, uniformity of the the size-based externality for middlemen is positive. However, the emergence of a Condorcet-like cycle in the economy prevents the emergence of the desired stability.

Example 4.16 Let $N = \{1,2,3,4,5\}$ and let $\Gamma = \{12,15,23,34,45\}$. Furthermore, let $\alpha_c = \alpha = 2$ for all potential middlemen $c \in \mathcal{K}(\Sigma(\Gamma)) = N$. Let the utility levels for each simple

[15]Here we introduce the following shorthand notation in the form of triples ijk to denote a feasible multilateral interaction consisting of the three agents i, j, and k where i acts as a middleman. Similarly we use the quadruplet $ijkl$ to describe a four-agent interaction with middleman i.

activity be given by $u_i(ii) = 0$ for all $i \in N$, $u_1(12) = u_2(23) = u_3(34) = u_4(45) = 2$, $u_1(15) = u_2(12) = u_3(23) = u_4(34) = u_5(45) = 10$ and $u_5(15) = -1$. The utility levels in all possible multilateral interactions are computed in a straightforward manner from the linear size-based externality formulation: $u_5(125) = 1$, $u_1(213) = u_2(324) = u_3(435) = u_4(514) = 4$, $u_5(514) = 11$, $u_1(514) = u_2(125) = u_3(213) = u_4(324) = u_5(435) = 12$, and $u_1(125) = u_2(213) = u_3(324) = u_4(435) = 14$. One can easily check that also in this example there is no stable multilateral outcome. ♦

We conclude from these three examples that size-based externalities prevent the emergence of a stable multilateral outcomes if (1) there are non-uniform externalities, or (2) there are negative size-based externalities, or (3) there are cycles in Γ. However, if these three conditions are ruled out, stability can be established.

Theorem 4.17 *Let $\mathbb{E} = (N, \Delta, u)$ be a network economy where u exhibits positive size-based externalities with $\alpha_c > 0$ for all potential middlemen $c \in \mathcal{K}(\Gamma)$. If Γ is acyclic, then $\mathbb{E}$ admits a stable multilateral outcome.*

A proof of this existence result can be found in Appendix C.

This assertion cannot be strengthened to the case of strong stability. The next example devises a simple case satisfying the conditions of Theorem 4.17 in which no strongly stable multilateral outcome can be constructed. Thus, in the presence of these externalities only economies with "open" multilateral economic activities can achieve stability.

Example 4.18 Let $N = \{1,2,3\}$ with $\Gamma = \{12, 23\}$ and $\Sigma(\Gamma) = \{213\}$. We consider the hedonic utility profile with size-based externalities generated by $\alpha = 2$ and $u_1(11) = u_3(33) = 0$, $u_2(22) = -4$, $u_1(12) = -1$, $u_2(23) = -3$, and $u_2(12) = u_3(23) = 1$. Hence, $u_1(213) = -1+2 = 1$, $u_2(213) = 1 - 3 + 2 = 0$, and $u_3(213) = 1 + 2 = 3$.
We now check that in this economy there is no strongly stable multilateral outcome: $\{11, 23\}$ is not stable since agent 1 wants to join agent 2 in the multilateral interaction 213 and its middleman, agent 2, agrees; $\{12, 33\}$ is not stable since IR is not satisfied for agent 1; $\{213\}$ is not strongly stable since its middleman, agent 2, prefers 12 over 213 and thus severs the participation of agent 3; and $\{11, 22, 33\}$ is not stable since agents 2 and 3 prefer the bilateral interaction 23 over being autarkic.
Although there is no strongly stable multilateral outcome in this network economy, multilateral interaction $\{213\}$ forms a stable one. ♦

What is clear from our analysis is that to render more complex economic outcomes stable for any distribution of preferences, the underlying network must satisfy a more complex set of properties. In the case of bilateral interactions, a necessary and sufficient characteristic is summarised in the rule of binary role assignment among economic agents.

Whereas for stability with multilateral interactions, these conditions are sensitive with respect to the presence of externalities within a multilateral interaction and the discretionary power of the middleman to sever just one of her existing interactions and not any others.

Our analysis reveals that when the middlemen have discretionary powers, the necessary and sufficient conditions for stability are more stringent, *i.e.*, more classes of cyclical network structures need to be ruled out. Whereas all even cycles are suited to support universally stable bilateral equilibria, only those even cycles that contain a multiple of 3 agents are also suited to support universally stable equilibria of multilateral interactions. This additional restriction points at the need for greater complexity in the institutional rules. We give a few examples of network structures that comply with these rules. Among these examples is that of a strict social hierarchy. We show that when externalities are allowed, we cannot find more relaxed institutional rules than the ones embedded in the hierarchy that are sufficient to guarantee the weaker form of stability of an outcome with multilateral interactions. We thus provide some evidence for the necessary co-evolution of complexity between institutional rules and economic interactions.

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Appendices

A Proof of Theorem 3.5

Here we show the necessary and sufficient conditions for the network structure to support universally bilateral stability. In Lemma 1, we establish a parallel with existing notions in the one-to-one matching literature.

Lemma 1 *Consider a bilateral economy* $\mathbb{E}^m = (N, \Delta^m, u^m)$. *Let the network structure* Γ *be bipartite in the sense that there exists a partitioning* $\{N_1, N_2\}$ *of* N *such that*

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}.$$

Then there exists a corresponding marriage problem (Gale and Shapley, 1962) such that a stable matching in the marriage problem corresponds to a stable bilateral outcome in the bilateral economy $\mathbb{E}^m$.

Proof. A marriage problem as introduced by Gale and Shapley (1962) consists of two finite and disjoint sets of players M and W. Each agent $m \in M$ has complete and transitive preferences, $\succeq^M_m$, over $W \cup \{m\}$ and each agent $w \in W$ has complete and transitive preferences, $\succeq^W_w$, over $M \cup \{w\}$. A matching is a function $\mu : M \cup W \to M \cup W$ of order two, i.e., $\mu(\mu(i)) = i$, $\mu(m) \in W \cup \{m\}$ and $\mu(w) \in M \cup \{w\}$. A matching μ is stable if there is no (a) player $m \in M$ or $w \in W$ who prefers to be matched to herself than to her partner in μ, or (b) pair of distinct players (m, w) who are not matched by μ and $w \succeq^M_m \mu(m)$ and $m \succeq^W_w \mu(w)$. Notice that conditions (a) and (b) correspond to conditions IR and PS of Definition 3.3, respectively.

Consider a bilateral economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ with a bipartite network structure Γ such that there exists a partitioning $\{N_1, N_2\}$ of N with

$$\Gamma \subseteq N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}.$$

Let $\widetilde{\Gamma} = N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}$. Next consider utility profile $\tilde{u}^m : \widetilde{\Gamma} \cup \Omega \to \mathbb{R}$ such that for all agents $i \in N$ and all matchings ij that satisfy the bipartite property but are *not* feasible, i.e., $ij \in \widetilde{\Gamma} \setminus \Gamma$, we set $\tilde{u}_i{}^m(ij) < u^m_i(ii)$, and for all matchings $ij \in \Delta^m$, we set $\tilde{u}^m = u^m$. Clearly, $\tilde{u}^m$ represents complete and transitive preferences on $\widetilde{\Gamma} \cup \Omega$.

Let $M = N_1$, $W = N_2$, and let preference profiles $\succeq^M$ and $\succeq^W$ be represented by hedonic utility functions $\phi^M_i : W \cup \{m\} \to \mathbb{R}$ with $\phi^M_i(N_i(ij)) = \tilde{u}_i(ij)$ for all $i \in M$ and all $ij \in \widetilde{\Gamma} \cup \Omega$ and $\phi^W_k(N_k(kl)) = \tilde{u}_k(kl)$ for all $k \in W$ and all $kl \in \widetilde{\Gamma} \cup \Omega$. The tuple $(M, W, \succeq^M, \succeq^W)$ defines a marriage problem.

Suppose μ^* is a stable matching in the marriage problem $(M, W, \succeq^M, \succeq^W)$. Consider, a bilateral outcome π^* in economy $\mathbb{E}$ such that $N_i(\pi^*(i)) = \mu^*(i)$ for all $i \in N$. Notice that $\pi^* \in \Delta^m$ follows from the stability of μ^*, which implies that for all $i \in M \cup W$, $\mu^*(i) \in N_i(\Delta^m)$, otherwise there is a contradiction to the stability of μ^* as there are two distinct players $k \in M$ and $l \in W$ with $\mu^*(k) = l$ and $kl \notin \Gamma$ such that k and l each prefer to be matched to themselves than to each other, i.e. $k \succeq^M_k l$ and $l \succeq^W_l k$ given by the construction of $\tilde{u}$, ϕ^M, and ϕ^W.

Lastly, we show that the stability of the matching function μ^* in the marriage problem implies the stability of the bilateral outcome π^* in the bilateral economy (N, Δ^m, u^m). The

proof follows by contradiction. Suppose the matching μ^* is stable and the bilateral outcome π^* is not stable. Therefore either IR or PS of Definition 3.3 must be violated.
Suppose, first, that IR does not hold and that there is an agent $i \in N$ such that $u_i(\pi^*) < u_i(ii)$. By construction, this implies that there is a player $i \in M$[16] such that $i \succeq_i^M \mu(i)$, which establishes a contradiction to the stability of μ^*.
Next, suppose that PS does not hold and that there are two distinct agents $i \in N_1$ and $j \in N_2$ with $ij \in \Gamma$ such that $u_i(ij) > u_i(\pi^*)$ and $u_j(ij) > u_j(\pi^*)$. By construction this implies that there are two distinct agents $i \in M$ and $j \in W$ with $\mu^*(i) \neq j$ such that $j \succeq_i^M \mu^*(i)$ and $i \succeq_j^W \mu^*(j)$ which contradicts to the stability of μ^*. ■

Proof of Theorem 3.5

If: Consider a bilateral economy $\mathbb{E}^m = (N, \Delta^m, u^m)$. Let the network structure Γ be bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that

$$\Gamma \subseteq N_1 \otimes N_2 = \{ \, ij \mid i \in N_1 \text{ and } j \in N_2 \, \} .$$

For any preference profile u^m, we can obtain a corresponding marriage problem as shown in Lemma 1. The existence of a stable matching in any marriage problem is shown by means of a constructive proof of Gale and Shapley (1962) and by means of a non-constructive proof in Sotomayor (1996). By analogy, this proves the existence of a stable bilateral outcome in bilateral economy $\mathbb{E}^m$ for any preference profiles u^m, given network structure Γ.

Only If: We show that if the network structure is not bipartite, there exists a preference profile for which there is no stable bilateral outcome in a bilateral economy.
Consider bilateral economy $\mathbb{E}^m = (N, \Delta^m, u^m)$ with $N = \{i, j, k\}$, and network structure $\Gamma = \{ij, ik, jk\}$. Consider the following preference profile: $u_i(ij) = u_j(jk) = u_k(ik) = 2$, $u_i(ik) = u_j(ij) = u_k(jk) = 1$, and $u_l(ll) = 0$ for all $l \in \{i, j, k\}$. It is easy to see that there is no stable bilateral outcome in this economy. For example, consider the outcome $\pi(i) = \pi(j) = ij$ and $\pi(k) = kk$. It is not stable because pairwise stability is not satisfied: $u_k(jk) > u_k(kk)$ and $u_j(jk) > u_j(ij)$. Similarly, one can show that no other bilateral outcome is stable.

This completes the proof of Theorem 3.5

B Proofs of Theorems 4.7 and 4.9

The following Lemma states an intermediate result that is required for the proof of the necessary and sufficient condition for the network structure to support universal strong stability in a network economy without any externalities.
Throughout we let $\mathbb{E} = (N, \Delta, u)$ be some network economy. As before let $\Delta^m = \Omega \cup \Gamma$ be a simple interaction structure on N and let $u \in \mathcal{U}$ be an arbitrary profile of utility functions, we denote by

$$B_i(\Delta^m, u) = \{ \, j \in N \mid ij \in \Delta^m \text{ and } u_i(ij) \geqslant u_i(ik) \text{ for all } k \in N \text{ with } ik \in \Delta^m \, \} \tag{13}$$

the *set of most preferred partners* of agent i for all $i \in N$.[17]

[16]Here we assume, without loss of generality, that $i \in M$. If we were to assume, instead, that $i \in W$ the argument follows analogously.
[17]Here $i \in B_i(\Delta^m, u)$ refers to agent i preferring to remain in autarky over the state of interaction with another agent.

Lemma 2 *Let the network structure* Γ *be acyclic. Then there is a pair of agents* $i,j \in N$ *with* $i \neq j$ *such that* $j \in B_i(\Delta^m,u)$ *and* $i \in B_j(\Delta^m,u)$.

Proof. If there is some agent $i \in N$ with $i \in B_i(\Delta^m,u)$ the assertion is obviously valid. Next assume that for every agent $i \in N$ it holds that $i \notin B_i(\Delta^m,u)$ and the second part of the assertion is not true. Then for all agents $i,j \in N$ with $i \neq j$ such that $j \in B_i(\Delta^m,u)$ it holds that $i \notin B_j(\Delta^m,u)$. Consider agent $i \in N$ and without loss of generality we may assume that the set of most preferred agents is a singleton, i.e., $B_i(\Delta^m,u) = \{j\}$. So, it must hold that $j \neq i$. Next, consider the set of most preferred partners of agent j. Without loss of generality we again may assume that B_j is a singleton, say $B_j(\Delta^m,u) = \{k\}$. It must again hold that $k \notin \{i,j\}$. Subsequently, consider the set of most preferred partners of agent k. Without loss of generality we again assume uniqueness, say $B_k(\Delta^m,u) = \{l\}$. It must be that $l \notin \{j,k\}$, moreover $l \neq i$ otherwise Γ contains a cycle. Hence, $l \notin \{i,j,k\}$. By continuing this process in a similar fashion, given that the player set N is finite, we construct a cycle. Therefore, we have established a contradiction. ■

Proof of Theorem 4.7

If: Consider a separable network economy $\mathbb{E} = (N,\Delta,u)$ such that $u \in \overline{\mathcal{U}}$ exhibits no externalities and is superadditive. We consider two separate cases: (I) when Γ does not contain any cycle; and (II) when Γ contains a cycle with an even number of connected agents that is a multiple of 3.
Let $S \subseteq N$ be some subset of economic agents. Then we denote by

$$\Gamma(S) = \Delta^m \cap \{ij \mid i,j \in S\}$$

the network structure and autarkic positions restricted to the subset S. In addition we use the operator $\oplus$ to denote an addition of an interaction to a given (partial) multilateral outcome, e.g. given the partial multilateral outcome $\Lambda = \{\{ijk\},\{ll\}\}$, $\Lambda \oplus \{ih\} = \{\{ijhk\},\{ll\}\}$. Finally, we slightly abuse notation and given a (partial) a multilateral outcome Λ, we denote by $N(\Lambda)$ all agents that are part of this outcome, i.e. they are part of an autarky, or bilateral, or multilateral interaction in Λ. Using these auxiliary notations we proceed with the proof of the two cases.

CASE I: Suppose Γ is acyclic.
We now devise an algorithm to construct a stable multilateral outcome in the economy $\mathbb{E}$ introduced above. This construction consists of several steps and collects agents in various multilateral interactions such that there are no possibilities for profitable deviations of all partners involved in the deviation.
We initiate the algorithm by setting N the set of agents, $\Gamma_1 = \Delta^m$ (the set of links that can be used in the construction of the outcome), $\Lambda_1 = \varnothing$ is a partial multilateral outcome and $K_1 = \varnothing$ is the set of agents who are active in outcome Λ and can act as middlemen. We now proceed by constructing the desired strongly stable multilateral outcome in a stepwise fashion:

Let N, $\Gamma_k \neq \emptyset$, Λ_k, K_k be given for k. We now proceed by constructing these elements for step $k+1$.

Take two agents $i \in N$ and $j \in N$ (notice that it is possible for $i = j$) such that $i \in B_j(\Gamma_k,u)$ and $j \in B_i(\Gamma_k,u)$. Such two agents exist for any non-empty $\Gamma_k \subseteq \Gamma$ by Lemma 2.

If $i = j$, then we define

$$\Lambda_{k+1} = \Lambda_k \cup \{ij\}; \tag{14}$$
$$\Gamma_{k+1} = \Gamma_k \setminus L_i(\Delta^m) ; \tag{15}$$
$$K_{k+1} = K_k. \tag{16}$$

Thus, in (14), we add the autarky $\{ii\}$ to the partial outcome Λ_k. In (15) we update the set of available interactions in Γ_k by eliminating all interactions that involve agent i. Last, we do not update the set of potential middlemen in the outcome Λ_{k+1} as the only new agent in this outcome, agent i, cannot add another link without exiting the autarkic state.
Subsequently we proceed to step $k + 1$ in our construction process.

If $i \neq j$ and $i \notin K_k$ and $j \notin K_k$, then we define

$$\Lambda_{k+1} = \Lambda_k \cup \{ij\}; \tag{17}$$
$$\Gamma_{k+1} = \Gamma_k \setminus \Gamma\left(N(\Lambda_{k+1})\right) ; \tag{18}$$
$$K_{k+1} = K_k \cup \{i,j\}. \tag{19}$$

Thus, in (17), we add the bilateral interaction $\{ij\}$ to the partial outcome Λ_k. In (18) we update the set of interactions Γ_k by eliminating all links among agents who are already part of the outcome Λ_{k+1}, i.e. these are the bilateral interactions, and the autarkic relations of agents i and j, and all interactions of i and j with any other agent who is part of the outcome Λ_k[18]. Last, in (19) we update the set of potential middlemen in the outcome Λ_{k+1} by adding both agents i and j as they can add interactions to the existing one.
Subsequently we proceed to step $k + 1$ in our construction process.

If $i \neq j$ and $i \notin K_k$ and $j \in K_k$, and $u_j(\Lambda_l \oplus \{ij\}) \leq u_j(\Lambda_k)$, then we define

$$\Lambda_{k+1} = \Lambda_k;$$
$$\Gamma_{k+1} = \Gamma_k \setminus \{ij\} ;$$
$$K_{k+1} = K_k.$$

This is the case when an agent wants to join a multilateral interaction but the middleman of this interaction is better-off if the interaction is not added. Thus, the only update is to eliminate the non-desirable interaction from the middleman's point of view from set of possible interactions to be considered in the next step.
We proceed to step $k + 1$ in our construction process.

If $i \neq j$ and $i \notin K_k$ and $j \in K_k$, and $u_j(\Lambda_l \oplus \{ij\}) > u_j(\Lambda_k)$, then we define

$$\Lambda_{k+1} = \Lambda_k \oplus \{ij\}; \tag{20}$$
$$\Gamma_{k+1} = \Gamma_k \setminus \{L_i(\Delta^m) \text{ for all } i \in N_j(\Lambda_{k+1})\} ; \tag{21}$$
$$K_{k+1} = K_k \setminus N_j(\Lambda_k). \tag{22}$$

This is the case when an agent wants to join a multilateral interaction and the middleman of this interaction is better-off when the interaction is added. Thus in (20) we add the interaction $\{ij\}$ to the existing multilateral or bilateral interaction in which agent j is involved in the partial outcome Λ_k. In (21) we remove from future consideration all interactions of

[18]Therefore, it is not possible for both $i \in K_k$ and $j \in K_k$ at any step k of the algorithm.

all agents with whom j is connected because those agents cannot add any new interaction without deleting the one with j. For the same reason, we update the set of possible middlemen in (22) by removing all agents with whom j is connected in Λ_k. This is only important if j is involved in a bilateral interaction in the outcome Λ_k.

We proceed through the procedure until for some $k = \bar{k}$ we arrive at the situation that $\Gamma_{\bar{k}} = \varnothing$. (Note that such a $\bar{k} \leqslant |\Gamma|$ always exists.) Now consider $\Lambda^\star = \Lambda_{\bar{k}}$. First, since the procedure devised above assigns every agent to either an autarkic activity, a bilateral interaction, or a multilateral interaction, $\Lambda^\star$ is a multilateral outcome. Furthermore, each constructed interaction in $\Lambda^\star$ is based on either the optimality of an autarkic interaction, the optimality of a bilateral interaction, or the optimality of adding an interaction for a middleman. In the latter case, the non-externality and superadditivity properties of the hedonic utilities imply that the utilities generated in the constructed multilateral interactions in $\Lambda^\star$ are maximal under the imposed restrictions as well. Finally, this also guarantees that the middleman of multilateral interaction $G \in \Sigma(\Gamma) \cap \Lambda^\star$ does not have any incentives to break any relationships with members $i \in N(G)$. This implies, therefore, that the constructed multilateral outcome $\Lambda^\star$ is indeed strongly stable as required.
This concludes the Proof of Case I.

Case II: Suppose Γ contains a cycle $C = (i_1, \dots, i_m)$ of length $m = 1 = 6s$ for some $s \in \mathbb{N}$. Depending on the utility profile, we distinguish two sub-cases.

Case II.a: First, consider a utility function $u_i \in \overline{\mathcal{U}}$ which satisfies superadditivity and the non-externality property, such that either (a) there exists an agent i_k with $k = 1, \dots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$; or (b) there are two consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$; or (c) there is a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 2, \dots, m-1$ and $j \notin C$ such that $j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, we can use the algorithm described in Case I to construct a strongly stable assignment since this utility profile ensures that in any subset $S \subseteq N$ there is an agent $i \in S$ such that $i \in B_i(\Delta^m|_S, u)$, or there is a pair of distinct agents $i, j \in S$ such that $j \in B_i(\Delta^m|_S, u)$ and $i \in B_j(\Delta^m|_S, u)$ where $\Delta^m|_S$ is the restriction of Δ^m on the agent set S. Thus, the property of Lemma 2 holds for such preference profiles.

Case II.b: Lastly, consider a profile of utility functions $u_i \in \overline{\mathcal{U}}$ such that there is no agent i_k with $k = 1, \dots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$, or there are no consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$, nor is there a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 1, \dots, m-1$ and $j \notin C$ such that $i_j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, without loss of generality, we may assume that $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1}) \leqslant u_{i_k}(i_k i_{k-1} i_{k+1})$, $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) \leqslant u_{i_k}(i_k i_{k-1} i_{k+1}) < u_{i_k}(i_k, i_{k+1})$, or $u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$.
Suppose, the profile of utility function is $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1}) \leqslant u_{i_k}(i_k i_{k-1} i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$. Then, a partial outcome $\Lambda^\star$ can be introduced that consists of exactly $2 \times s$ multilateral interactions of the type

$$\{\, \{i_2 i_1 i_3\}, \{i_5 i_4 i_6\}, \dots, \{i_{m-2} i_{m-3} i_{m-1}\} \,\} \subseteq \Lambda^\star.$$

Next, all other agents are assigned following the algorithm presented in Case I. Thus, we

have constructed a (complete) multilateral outcome $\Lambda^\star$, which furthermore is strongly stable: all agents who are not in an interaction with their most preferred partner, have their most preferred partner in an interaction with her own most preferred partner. This implies that they have no incentive to sever their links; moreover, these agents are not in a bilateral interaction and, therefore, they cannot add an interaction without severing an existing interaction.
Suppose, the profile of utility function is $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) \leqslant u_{i_k}(i_k i_{k-1} i_{k+1}) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial multilateral outcome $\Lambda^\star$ can be introduced that consists of exactly $3 \times s$ matchings of the type

$$\{\{i_1 i_2\}, \{i_3 i_4\}, \ldots, \{i_{m-1} i_{m-2}\}\} \subseteq \Lambda^\star.$$

All other agents are assigned following the algorithm presented in Case I. Thus, we have constructed a (complete) multilateral outcome t $\Lambda^\star$, which furthermore is strongly stable: all agents who are not in an interaction with their most preferred partner, have their most preferred partner in an interaction to her own most preferred partner. This implies that they have no incentive to sever their interaction. Moreover, these agents are better-off in a bilateral interaction than in a multilateral interaction acting as middlemen. Therefore, they will not add an interaction.
Last, suppose that the profile of utility function is $u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$. Then, a partial multilateral outcome $\Lambda^\star$ can be introduced that consists of exactly $m-1$ autarkic agents

$$\{\{i_1 i_1\}, \{i_2 i_2\}, \ldots, \{i_{m-1} i_{m-1}\}\} \subseteq \Lambda^\star.$$

All other agents are assigned following the algorithm presented in Case I. Thus, we have constructed a (complete) multilateral outcome $\Lambda^\star$, which furthermore is strongly stable: all along the cycle agents are autarkic as the only partner whom they prefer to being autarkic prefers to be autarkic himself than to be in an interaction with them.
This completes the proof of Case II.

Only if: Let $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ be a feasible activity structure and let $\overline{\mathcal{U}}$ be the collection of all superadditive and non-externality hedonic utility profiles. Let there be a strongly stable multilateral outcome in the network economy (N, Δ, u) for all $u \in \overline{\mathcal{U}}$. We show by contradiction the necessity of the condition that Γ contains no cycles, or that if it contains a cycle, it is a cycle with an even number of connected agents which is also a multiple of 3. We discuss two cases: the first case is when the length of the cycle is even but not a multiple of three, and the second one is when the length is odd.

Case I: Suppose that the network structure Γ contains a cycle $C = (i_1, i_2, \ldots, i_m)$ with $i_k, i_{k+1} \in \Gamma$ for all $k = 1, \ldots, m-1$ and $m \geqslant 4$ and $m-1$ is an even number which is not a multiple of 3.
Now, consider a utility profile $u \in \overline{\mathcal{U}}$ such that $u_j(j i_k) < u_j(jj)$, $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k, i_{k+1}) < u_{i_k}(i_k i_{k-1} i_{k+1})$ for all $k = 1, \ldots, m-1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(\Gamma) \setminus \{i_{k-1}, i_{k+1}\}$. Let $\Lambda^\star$ be a strongly stable multilateral outcome in this network economy. Note that in the strongly stable outcome $\Lambda^\star$ the largest number of agents located along the cycle who can form a multilateral interaction that satisfies IR is three and that all of the agents in such a multilateral interaction are located along the cycle. In addition, since the length of the cycle is not a multiple of three, it must be that in $\Lambda^\star$ at least one agent is

autarkic or at least two agents are in a bilateral interaction. We consider two sub-cases.

Case I.a: First, suppose that $i_k i_k \in \Lambda^\star$ for some $k = 1, \dots, m-1$. Since $\Lambda^\star$ is a strongly stable outcome, the individual rationality condition is satisfied for all agents in N. Hence, agent i_{k-1} is in a state of autarky or connected to agent i_{k-2} either in through the bilateral interaction $g' = \{i_{k-1} i_{k-2}\}$, or the multilateral interaction $g'' = \{i_{k-2} i_{k-1} i_{k-3}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, and $i_{-2} = i_{m-3}$. In all three cases the PS condition is violated: $u_{i_k}(i_{k-1} i_k) > u_{i_k}(i_k i_k)$ and $u_{i_{k-1}}(i_{k-1} i_k) > u_{i_{k-1}}(g'') = u_{i_{k-1}}(g') > u_{i_{k-1}}(i_{k-1} i_{k-1})$. Therefore the strong stability of $\Lambda^\star$ implies that $\{i_k i_k\} \notin \Lambda^\star$ for any $i_k \in C$.

Case I.b: Next, suppose that strongly stable multilateral outcome $\Lambda^\star$ contains a bilateral interaction $\{i_{k-1}, i_k\}$. Then, agent i_{k-2} is connected to agent i_{k-3} either through the bilateral interaction $g' = \{i_{k-2} i_{k-3}\}$, or the multilateral interaction $g'' = \{i_{k-3} i_{k-2} i_{k-4}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, $i_{-2} = i_{m-3}$, and $i_{-3} = i_{m-4}$.[19] In all cases the no blocking condition PS* is violated: $u_{i_{k-2}}(i_{k-1} i_{k-2} i_k) > u_{i_{k-2}}(g') = u_{i_{k-2}}(g'')$ as the bilateral interaction $i_k i_{k-2} \notin \Gamma$ and $u_{i_{k-1}}(\{i_{k-1} i_{k-2} i_k\}) \geqslant u_{i_{k-1}}(i_{k-1} i_k)$ with $k_{-1} = m-2$ due to superadditivity.
Hence, when Γ contains a cycle with an even number of connected agents which is not a multiple of three, there are such utility profiles that satisfy superadditivity and non-externality properties, for which there is no stable assignment in the network economy.

Case II: Now suppose that the network structure Γ contains a cycle $C = (i_1, i_2, \dots, i_m)$ with $i_k, i_{k+1} \in \Gamma$ for all $k = 1, \dots, m-1$ and $m \geqslant 4$ and $m-1$ is an odd integer.
Now, consider a utility profile $u \in \overline{\mathcal{U}}$ such that $u_j(j i_k) < u_j(jj)$, $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k i_{k-1} i_{k+1}) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(\Gamma) \setminus \{i_{k-1}, i_{k+1}\}$. Let $\Lambda^\star$ be a strongly stable multilateral outcome in this network economy. Note that in the strongly stable outcome $\Lambda^\star$ the largest number of agents located along the cycle that can form a multilateral interaction and that satisfies the IR condition is three. In addition, since the length of the cycle is odd, in the outcome $\Lambda^\star$ there must be at least one agent who is autarkic or at least three agents who are in a multilateral interaction. We consider two sub-cases.

Case II.a: First, suppose that $i_k i_k \in \Lambda^\star$ for some $k = 1, \dots, m-1$. Similar to Case I.a, we can show that the PS condition must be violated as $u_{i_k}(i_{k-1} i_k) > u_{i_k}(i_k i_k)$ and $u_{i_{k-1}}(i_{k-1} i_k) > u_{i_{k-1}}(\Lambda^\star) \geq u_{i_{k-1}}(i_{k-1} i_{k-1})$. Since $\Lambda^\star$ is strongly stable, then it cannot be that $\{i_k i_k\} \in \Lambda^\star$ for some $i_k \in C$.

Case II.b: Lastly, suppose that the multilateral interaction $\{i_k i_{k-1} i_{k+1}\} \in \Lambda^\star$ for some $k = 1, \dots, m-1$ with $k_0 = i_m$ and $k_{m+1} = i_1$. In this case the RP condition is violated as $u_{i_k}(i_k i_{k-1} i_{k+1}) < u_{i_k}(i_k i_{k+1})$. Since $\Lambda^\star$ is strongly stable, then it cannot be that $\{i_k i_{k-1} i_{k+1}\} \in \Lambda^\star$ for some $i_{k-1}, i_k, i_{k+1} \in C$.
Hence, when Γ contains a cycle with an odd number of connected agents, there are such utility profiles that satisfy superadditivity and non-externality properties, for which there is no strongly stable multilateral outcome in the network economy.

This completes the proof of Theorem 4.7.

Proof of Theorem 4.9

If: Consider a separable network economy $\mathbb{E} = (N, \Lambda, u)$ such that $u \in \overline{\mathcal{U}}$ exhibits no externalities and is superadditive. We consider two cases: (I) when Γ does not contain any

[19] Recall that Case I.a rules out that $\{i_{k-2}, i_{k-2}\} \in \Lambda^\star$.

cycle; (II) when Γ contains a cycle with a number of connected agents that is a multiple of 3.

Case I: Suppose that Γ is acyclic. Since strong stability implies stability, the proof of Case I follows the steps in Case I of the proof of Theorem 4.7.

Case II: Suppose that Γ has a cycle $C = (i_1, \dots, i_m)$ with $m \geqslant 4$ and $m - 1 = 3s$ for some $s \in \mathbb{N}$. Depending on the utility profile, we distinguish two sub-cases analogous to those discussed in the proof of the Theorem 4.7.

Case II.a: First, consider a utility function $u_i \in \overline{\mathcal{U}}$ which satisfies superadditivity and the non-externality properties, such that either (a) there exists an agent i_k with $k = 1, \dots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$; or (b) there are two consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$; or (c) there is a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 2, \dots, m-1$ and $j \notin C$ such that $j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, we can use the algorithm described in Case I to construct a stable multilateral outcome since the utility profile ensures that in any of the three cases described above, we can identify agents that fit the requirements stated in Lemma 2.

Case II.b: Next, consider a profile of utility functions $u_i \in \overline{\mathcal{U}}$ which satisfies superadditivity and non-externality such that there is no agent i_k with $k = 1, \dots, m-1$ such that $i_k \in B_{i_k}(\Delta^m, u)$, or there are no consecutive agents along the cycle $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_{i_{k-1}}(\Delta^m, u)$, nor is there a pair of agents one of whom is on the cycle and the other not, *i.e.*, $i_k \in C$ for some $k = 1, \dots, m-1$ and $j \notin C$ such that $i_j \in B_{i_k}(\Delta^m, u)$ and $i_k \in B_j(\Delta^m, u)$. Then, without loss of generality, we may assume that $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1})$, or $u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$.
Suppose, the profile of utility functions is $u_{i_k}(i_k i_k) \leqslant u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$. Then, a partial multilateral outcome $\Lambda^\star$ can be introduced that consists of exactly s multilateral interactions of the type

$$\{\, \{i_2 i_1 i_3\}, \{i_5 i_4 i_6\}, \dots, \{i_{m-2} i_{m-3} i_{m-1}\} \,\} \subseteq \Lambda^\star.$$

Now, all other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) multilateral outcome $\Lambda^\star$, which furthermore is stable: all agents who are not in interaction with their most preferred partner, have their most preferred partner in interaction with her own most preferred partner. This implies that they have no incentive to sever their links; moreover, these agents are not in a bilateral interaction and, therefore, they cannot add a link without severing an existing link. In addition, due to the superadditivity of the utility profile, all middlemen prefer to be linked in a multilateral interaction than to be autarkic.
Last, suppose that the profile of utility function is given by $u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$. Then, a partial outcome $\Lambda^\star$ can be introduced that consists of exactly $m-1$ autarkic agents

$$\{\, \{i_1 i_1\}, \{i_2 i_2\}, \dots, \{i_{m-1} i_{m-1}\} \,\} \subseteq \Lambda^\star.$$

All other agents are linked following the algorithm presented in Case I. Thus, we have constructed a (complete) assignment $\Lambda^\star$, constituting a stable outcome: All agents on the cycle are autarkic as the only partner whom they prefer to being autarkic prefers to be

autarkic himself than to be matched with them.
This completes the proof of Case II.

Only if: Let $\Delta = \Omega \cup \Gamma \cup \Sigma(\Gamma)$ be a feasible activity structure and let $\overline{\mathcal{U}}$ be the collection of all superadditive and non-externality hedonic utility profiles. We show by contradiction the necessity of the condition that Γ contains no cycles or if it contains a cycle it is a cycle with a number of connected agents equal $m \geqslant 4$ with $m-1 \neq 3s$ with $s \in \mathbb{N}$.
Let there be a stable multilateral outcome in the network economy (N, Δ, u) for all $u \in \overline{\mathcal{U}}$. Let the network structure Γ contain a cycle $C = \{i_1, i_2, \dots, i_m\}$ with $i_k, i_{k+1} \in \Gamma$ for all $k = 1, \dots, m-1$ and $m \geqslant 4$ and $m-1 \neq 3s$ with $s \in \mathbb{N}$.
Now, consider a utility profile $u \in \overline{\mathcal{U}}$ such that $u_j(ji_k) < u)j(jj)$, $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k, i_{k+1}) < u_{i_k}(i_k i_{k-1} i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(\Gamma) \setminus \{i_{k-1}, i_{k+1}\}$. Let $\Lambda^\star$ be a stable multilateral outcome in this network economy. Note that in the stable outcome $\Lambda^\star$ the largest number of agents along the cycle that can form a multilateral interaction that satisfies the IR condition is three.
Since the length of the cycle is not a multiple of 3, in any assignment along the cycle there must be at least one agent who is autarkic, or at least two distinct agents who are in a bilateral interaction. We discuss these two sub-cases separately.

Case I: First, suppose that $i_k i_k \in \Lambda^\star$ for some $k = 1, \dots, m-1$. Since $\Lambda^\star$ is a stable outcome, the individual rationality condition is satisfied for all agents in N. Hence, agent i_{k-1} is in a state of autarky or connected to agent i_{k-2} either in the bilateral interaction $g' = \{i_{k-1} i_{k-2}\}$, or in the multilateral interaction $g'' = \{i_{k-2} i_{k-1} i_{k-3}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, and $i_{-2} = i_{m-3}$. In all three cases the PS condition is violated: $u_{i_k}(i_{k-1} i_k) > u_{i_k}(i_k i_k)$ and $u_{i_{k-1}}(i_{k-1} i_k) > u_{i_{k-1}}(g'') = u_{i_{k-1}}(g') > u_{i_{k-1}}(i_{k-1} i_{k-1})$. Since $\Lambda^\star$ is stable, then it cannot be that $\{i_k i_k\} \in \Lambda^\star$ for some $i_k \in C$.

Case II: Next, let the bilateral interaction $\{i_{k-1}, i_k\} \in \Lambda^\star$ for some $k = 1, \dots, m-1$ and $k_0 = m-1$. Then, agent i_{k-2} is connected to agent i_{k-3} either in the bilateral interaction $g' = \{i_{k-2} i_{k-3}\}$, or in the multilateral interaction $g'' = \{i_{k-3} i_{k-2} i_{k-4}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, $i_{-2} = i_{m-3}$, and $i_{-3} = i_{m-4}$. In all cases the no blocking condition PS* is violated: $u_{i_{k-2}}(i_{k-1} i_{k-2} i_k) > u_{i_{k-2}}(g') = u_{i_{k-2}}(g'')$ and $u_{i_{k-1}}(\{i_{k-1} i_{k-2} i_k\}) > u_{i_{k-1}}(i_{k-1} i_k)$ with $k_{-1} = m-2$ due to superadditivity.
Hence, when Γ contains a cycle with a number of connected agents not a multiple of three, there are such utility profiles that satisfy superadditivity and non-externality properties, for which there is no stable multilateral outcome in the network economy.

This completes the proof of Theorem 4.9.

C Proof of Theorem 4.17

Before we present the proof we state the following auxiliary results.

Lemma 3 *Let (N, Δ, u) be a network economy and let Γ be a network structure that contains no cycles. Then there do not exist two paths with distinct agents along each path between any two agents in N.*

Proof. The statement follows immediately from the fact that the network structure Γ contains no cycles. It is clear that if there were two distinct paths that connect two distinct agents, these two paths would constitute a cycle. ■

Lemma 4 *Let (N, Δ, u) be a network economy and let Γ be a network structure that contains no cycles. Then there exist at last two distinct agents in N who have exactly one link in Γ.*

Proof. It is easy to show that Lemma 4 also follows immediately from the fact that the network structure Γ contains no cycles and the finite number of agents in N. Suppose there is at most one agent in N who has exactly one link in Γ.[20] Take any agent $i \in N$ and suppose she has two links in Γ with agents j and k, respectively, where $j \neq k$. Since all agents but one have at least two links, agent j or k must have at least two links, too. Suppose, agent j has exactly two links with agents i and l where $l \neq i$ and $l \neq k$, otherwise there is a cycle in Γ. Similarly, agent l must have at least two links in Γ. Suppose agent l has exactly two links with agents j and m where $m \neq j$, $m \neq i$ and $m \neq k$ otherwise there is a cycle in Γ. Following the same logical steps one arrives at the conclusion that the absence of cycles in Γ and the finiteness of the agent set requires that there are at least two agents who have exactly one link. ■

Proof of Theorem 4.17

Let $\mathbb{E} = (N, \Delta, u)$ be a network economy such that u exhibits multiplicative size-based externalities such that $\alpha_c > 0$ for all potential middlemen $c \in \mathcal{K}(\Gamma)$. Suppose Γ contains no cycles. Without loss of generality suppose that there is a path in Γ connecting any two distinct agents in N.[21]
Next we re-label the argents to form a sequence that abides by the following rules:

1. Agents in the set N are labelled $1, 2, \ldots, N$ such that any agent with label k where $k = 2, 3, \ldots, N$, is connected to exactly one agent in the set $1, \ldots, k-1$. By Lemma 4, there are at least two agents in the set N who have exactly one link in Γ. Suppose these are agents i and j. Thus we can re-label $i = 1$ and $j = N$.

2. The length of the paths from agent 1 to any two consecutive agents in the sequence, $k-1$, k with $k = 2, \ldots, N$, i.e. $|p_{1k-1}|$ and $|p_{1k}|$ cannot differ by more than a unit where the path of the agent with the higher label is at least as long as the one of the agent with the lower label ($|p_{1k-1}| + 1 \geq |p_{1k}|$).

The proof now proceeds by induction. Suppose there are stable outcomes, Λ_{k-2}, Λ_{k-1} and Λ_k, in the network economies restricted to the first $k-2$, $k-1$ and k agents in the sequence and the links amongst them with $k \geq 3$, i.e., (N^s, Γ^s, u) with $N^s = \{1, \ldots, s\}$ and $\Gamma^s = \{ij \in \Gamma$ such that $i \in N^s$ and $j \in N^s\}$ for $s = k-2, k-1, k$.
Consider the network economy $(N^{k+1}, \Gamma^{k+1}, u)$ restricted to the first $k+1$ agents and the links amongst them, i.e., $N^{k+1} = \{1, \ldots, k+1\}$ and $\Gamma^{k+1} = \{ij \in \Gamma$ such that $i \in N^{k+1}$ and $j \in N^{k+1}\}$. Notice that by the construction of the sequence of agents, Γ^{k+1} differs from Γ^k only by the additional link of agent $k+1$ with exactly one agent in N^k. For ease of exposition, suppose that agent $k+1$ has a link with agent k in Γ^{k+1}; agent k has a link with

[20] Recall that we have ruled out the trivial case when there are agents who are not linked in Γ. Thus, all agents in N have at least one link in Γ.

[21] In other words we assume that the graph consists of a single component. This assumption goes without loss of generality as should there be more than one components in the graph, the reasoning presented below can be applied to each component separately. Since there is no link that connects individuals from different components, there are no externalities that need to be taken into account in the construction of a stable outcome either.

agent $k-1$ in Γ^k; and agent $k-1$ has a link with agent $k-2$ in Γ^{k-1}. In the discussion below, we point out how this restriction can be relaxed.

Case I: Suppose that under Λ_k agent k can add the link with agent $k+1$ without deleting all her links.[22]

If the utility that agent k can gain from becoming a middleman is at most the utility she would lose from the direct link with $k+1$ ($u_k(kk+1) \leq -\alpha_k$) or agent $k+1$ is as better off autarkic as he is in a multilateral interaction where agent k acts as a middleman with two other participants ($u_{k+1}(k+1k+1) \geq u_{k+1}(kk+1) + \alpha_k$), then $\Lambda_{k+1} = \Lambda_k \cup \{k+1k+1\}$ is a stable outcome in this economy.[23]

If, on the other hand, both agents k and $k+1$ are better-off by adding the interaction ($u_k(kk+1) > -\alpha_k$ and $u_{k+1}(kk+1) + \alpha_k > u_{k+1}(k+1k+1)$, then $\Lambda_{k+1} = \Lambda_k \oplus \{kk+1\}$ is stable where the operator $\oplus$ as defined above signifies that agent k has added the link with $k+1$ to her existing interactions in Λ_k. This is the case because all agents who are linked to k in Λ_k gain α_k in utility due to the size-based externality, thus, these agents would not want to deviate in Λ_{k+1} if they do not want to deviate in Λ_k where their utility is lower and have the same set of potential partners.[24]

Last, consider the case when agent k prefers to sever the link with $k-1$ and join $k+1$ in a bilateral interaction ($u_k(kk-1) < u_k(kk+1) < -\alpha_k$) and $k+1$ is better off in the bilateral interaction with k than in an autarky ($u_{k+1}(k+1k+1) < u_{k+1}(kk+1)$). Then the outcome $\Lambda_{k+1} = \Lambda_{k-1} \cup \{kk+1\}$ is stable. To see that recall that the outcome Λ_{k-1} is stable for all N^{k-1} agents and that by Lemma 3 the only link between the set of agents N^{k-1} and $\{k, k+1\}$ is the one between $k-1$ and k. These two players, however, cannot form a blocking pair as clearly the PS and PS* condition when k acts as a middleman are satisfied given the conditions on the utility function of agent k. The PS* condition when $k-1$ acts as a middleman must be satisfied since $\{k-1, k\} \in \Lambda_k$. This implies that either agent $k-1$ under Λ_{k-1} cannot act as a middleman, or that $u_{k-1}(k-1, k) < -\alpha_{k-1}$, hence agent $k-1$ does not want to add the link with k without severing all his existing links in Λ_{k-1}.[25]

Case II: Next, suppose that under Λ_k agent k cannot add the link with agent $k+1$ without deleting all her links. If agent k is at least as better off under the outcome Λ_k as she is in a bilateral interaction with $k+1$ ($u_k(\Lambda_k) \geq u_k(k, k+1)$) or if agent $k+1$ is at least as better-off autarkic than as he is in a bilateral interaction with agent k ($u_{k+1}(k+1, k+1) \geq u_{k+1}(k, k+1)$, then $\Lambda_{k+1} = \Lambda_k \cup \{k+1, k+1\}$ is a stable outcome in this economy. This is easy to see, since

[22]By construction this implies that the interaction $\{k-1k\} \in \Lambda_k$.

[23]Notice that here the assumption that agents $k-1$and k have only a link with agent $k-2$ in Γ^{k-1} and $k-1$ in Γ^k, respectively, goes without loss of generality. The same reasoning would hold if k is a middleman of a multilateral interaction with s members and the only amendment that would be necessary is to require that agent $k+1$ is as better off autarkic as in a multilateral interaction with k as a middleman and s other members ($u_{k+1}(k+1k+1) \geq u_{k+1}(kk+1) + \alpha_k s$).

[24]Notice again that the reasoning does not hinge on the assumption that k has a link with only one agent in Γ^k. Moreover, additional straightforward requirements on the ordering of the agents in the sequence can ensure that there are no agents with a label preceding that of $k+1$ who have a link with k and who prefer not to be linked to k in Λ_k but prefer to be linked with her in Λ_{k+1}.

[25]Here, the assumption that agent k has only one link and that is with agent $k-1$ who is preceding her in the sequence requires a clarification. Had agent k have also links with other agents whose labels follow k and precede $k+1$, then those agents would have been left autarkic in the stable outcome Λ_{k+1}. Recall that by the definition of the sequence all such agents would be equidistant from the origin, agent 1, as agent $k+1$, thus, all such agents would have had only one link in Γ^{k+1} and that would have been with agent k. Since k would sever all links to be in a bilateral interaction with $k+1$, those agents would remain autarkic with no potential partners to form links but k.

by construction agent $k+1$ has a link only with agent k and these two agents do not want to engage,[26] then the stability of Λ_k implies the stability of Λ_{k+1}.

Suppose, instead, that agent k prefers to sever her links in Λ_k to be in a bilateral interaction with $k+1$ ($u_k(\Lambda_k) < u_k(k,k+1)$ and $k+1$ prefers to be in a bilateral interaction with k than autarkic ($u_{k+1}(kk+1) < u_{k+1}(k+1,k+1)$. If agent $k-1$ is at least well-off under outcome Λ_{k-1} as in a multilateral interaction of size 3 with agent k acting as a middleman ($u_{k-1}(\Lambda_{k-1}) \geq u_{k-1}(k-1,k) + \alpha_k$) or the utility agent k gains from the direct link with agent $k-1$ is at most equal to the negative of the size-based externality she can generate as a middleman ($u_k(k-1,k) \leq -\alpha_k$), then $\Lambda_{k+1} = \Lambda_{k-1} \cup \{k,k+1\}$ is a stable outcome. That there are no blocking possibilities between $k-1$ and k is ensured by the stability of Λ_{k-1} and Λ_k, where agent k is either autarkic or in a multilateral interaction of which she is not the middleman (due to the fact that she has to sever all links in Λ_k to add a link with $k+1$), and the above restrictions on the utility profiles which dictate the satisfaction of all non-blocking conditions between players $k-1$ and k.[27]

Last consider the case where agent k prefers to sever her links in Λ_k to be in a bilateral interaction with $k+1$ ($u_k(\Lambda_k) < u_k(k,k+1)$ and $k+1$ prefers to be in a bilateral interaction with k than autarkic ($u_{k+1}(k,k+1) < u_{k+1}(k+1,k+1)$. In addition, let agent $k-1$ be better-off in a multilateral interaction of size 3 with agent k acting as a middleman than under Λ_{k-1} ($u_{k-1}(\Lambda_{k-1}) < u_{k-1}(k-1,k) + \alpha_k$) and the utility agent k gains from adding agent $k-1$ to the multilateral interaction is strictly positive ($u_k(k-1,k) > -\alpha_k$), then $\Lambda_{k+1} = \Lambda_{k-2} \cup \{k.k-1.k+1\}$ is a stable outcome. To see that notice that the only blocking possibility for $k+1$ is to the autarkic state which is ruled out by the preference profile and the fact that $k+1$ gains from the positive size-based externality when $k-1$ joins the multilateral interaction. The same analysis holds for the blocking possibility of agent k, which is ruled out by the preference profile specified above and that Λ_k is stable, thus, the IR is satisfied for all agents, including k.[28] In addition to the autarkic state, which is ruled out as a blocking possibility in a similar fashion as it is done for agents k and $k+1$, agent $k-1$ may have a blocking possibility with agent $k-2$ due to the failure of the PS or PS* condition when $k-2$ acts as a middleman.[29] These two conditions, however, are guaranteed

[26]Notice that in this case the assumption that agents $k-1$ and k have only one link in Γ^{k-1} and Γ^k, respectively, goes without loss of generality as no other agent who has a link with k can have a link with $k+1$ by Lemma 3.

[27]Similar to the discussion in footnote 21, had agent k have multiple links with agents whose labels follow hers, those agents would be autarkic in the stable outcome Λ_{k+1}.

[28]If there were other agents but $k+1$ who followed k and had a link with her, the construction of the stable outcome would have involved the addition to the multilateral interaction of all those agents who preferred to be members of the multilateral interaction than being autarkic and who earn sufficiently high utility to k for her to add the link. The remainder of the agents would stay autarkic in Λ_{k+1}. Such an outcome would be stable as neither the autarkic players nor those in the multilateral interaction whose label is higher than k have any other links in Γ^{k+1} but the one with k. As in the analysis provided in the main text, in this case, too, the only blocking possibility for k would be to sever all links but given the utility profile and the fact that Λ_k is stable, the IR condition is satisfied.

[29]The assumption that agent $k-1$ has only a link with $k-2$ in Γ^{k-1} can be relaxed in a similar fashion as the assumption concerning agent k. If there are agents who have labels higher than $k-1$ (other than k) and who have a link with $k-1$, then when $k-1$ severs his links with them, they may only participate in activities with other agents who are equidistant from the origin as $k+1$ (i.e. be in a bilateral interaction or a middleman of a multilateral interaction) given the rules by which the sequence is constructed or be autarkic. Notice that due to Lemma 3 the presence of $k+1$ does not present any further blocking possibilities for such agents than the ones present under Λ_k. Thus Λ_{k+1} can be augmented by including these agents in stable partial outcome of autarkies, bilateral, or multilateral interactions amongst them. Moreover, these agents do not have any links with agents in N^{k-2}, thus the stability of Λ^{k-2} holds through.

by the requirement that $u_{k-1}(\Lambda_{k-1}) < u_{k-1}(k-1,k)+\alpha_k$. Therefore, by the above discussion and the fact that Λ_{k-2} is a stable outcome, we have shown that Λ_{k+1} constitutes a stable outcome, too.

Finally to complete the proof by induction we show that the initial conditions for $k = 1,2,3$ are satisfied. The case when $k = 1$ is trivial as $\Lambda_1 = |11|$ is clearly stable. Consider $N^2 = \{1,2\}$ with $\Gamma^2 = \{12\}$. If $u_1(11) \geq u_1(12)$ or $u_2(22) \geq u_2(12)$, then $\Lambda_2 = \{\{11\},\{22\}\}$ is stable. Otherwise, if both agents prefer to be in a bilateral interaction than autarkic, then $\Lambda_2 = \{12\}$ is stable. Last, consider $N^3 = \{1,2,3\}$ with $\Gamma^2 = \{\{12\},\{23\}\}$. If all agents are at least as better-off in autarky as in any bilateral interaction, $u_1(11) \geq u_1(12)$ or $u_2(22) \geq u_2(12)$ and $u_2(22) \geq u_2(23)$ or $u_3(33) \geq u_3(23)$, then $\Lambda_3 = \{\{11\},\{22\},\{33\}\}$ is stable. In case at least one pair of agents who have a link prefer to be in a bilateral interaction than in autarky but the third agent prefers autarky than to be in a multilateral interaction, or the agent who may act as a middleman prefers not to add the link, we have the following stable outcome. If $u_1(11) < u_1(12)$ and $u_2(22) < u_2(12)$ and $u_3(33) \geq u_3(23) + \alpha_2$ or $u_2(23) \leq -\alpha_2$, then $\Lambda_3 = \{\{12\},\{33\}\}$ is stable. Similarly, if $u_3(33) < u_3(23)$ and $u_2(22) < u_2(23)$ and $u_1(11) \geq u_1(12)+\alpha_2$ or $u_2(12) \leq -\alpha_2$, then $\Lambda_3 = \{\{11\},\{23\}\}$ is stable. Finally, a multilateral interaction $\Lambda_3 = \{213\}$ would be stable in the following two cases: $u_1(11) < u_1(12)$ and $u_2(22) < u_2(12)$ and $u_3(33) < u_3(23) + \alpha_2$ and $u_2(23) > -\alpha_2$ or $u_3(33) < u_3(23)$ and $u_2(22) < u_2(23)$ and $u_1(11) < u_1(12) + \alpha_2$ and $u_2(12) > -\alpha_2$.

This completes the proof of Theorem 4.17.

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